Put answers on a separate piece of paper. Label each Section. Show all work for Free Response questions.

Quick Quiz for AP* Preparation: Sections 6.1–6.3

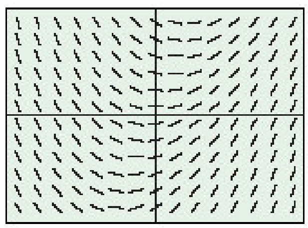
You should solve the following problems without using a graphing calculator.

1. Multiple Choice Which of the following differential equations would produce the slope field shown below?

(A) $\frac{dy}{dx} = y - 3x$ **(B)** $\frac{dy}{dx} = y - \frac{x}{3}$

(C) $\frac{dy}{dx} = y + \frac{x}{3}$ **(D)** $\frac{dy}{dx} = y + \frac{x}{3}$

(E) $\frac{dy}{dx} = x - \frac{y}{3}$



2. Multiple Choice If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y\,dy$ **(B)** $2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$

(C) $2\int_0^{\pi/4} \sin^2 y\,dy$ **(D)** $\int_0^{\pi/4} \sin^2 y\,dy$

(E) $2\int_0^{\pi/6} \sin^2 y\,dy$

3. Multiple Choice $\int x\,e^{2x}\,dx =$

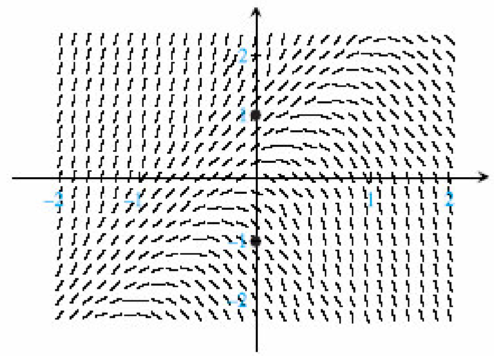
(A) $\frac{x\,e^{2x}}{2} - \frac{e^{2x}}{4} + C$ **(B)** $\frac{x\,e^{2x}}{2} - \frac{e^{2x}}{2} + C$

(C) $\frac{x\,e^{2x}}{2} + \frac{e^{2x}}{4} + C$ **(D)** $\frac{x\,e^{2x}}{2} + \frac{e^{2x}}{2} + C$

(E) $\frac{x^2\,e^{2x}}{4} + C$

4. Free Response Consider the differential equation $dy/dx = 2y - 4x$.

(a) The slope field for the differential equation is shown below. Sketch the solution curve that passes through the point (0, 1) and sketch the solution curve that goes through the point (0, −1).



(b) There is a value of b for which $y = 2x + b$ is a solution to the differential equation. Find this value of b. Justify your answer.

(c) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. It appears from the slope field that g has a local maximum at the point (0, 0). Using the differential equation, prove analytically that this is so.

Quick Quiz: Sections 6.4 and 6.5

You may use a graphing calculator to solve the following problems.

1. **Multiple Choice** The rate at which acreage is being consumed by a plot of kudzu is proportional to the number of acres already consumed at time t. If there are 2 acres consumed when $t = 1$ and 3 acres consumed when $t = 5$, how many acres will be consumed when $t = 8$?

   (A) 3.750 (B) 4.000 (C) 4.066 (D) 4.132 (E) 4.600

2. **Multiple Choice** Let $F(x)$ be an antiderivative of $\cos(x^2)$. If $F(1) = 0$, then $F(5) =$

   (A) -0.099 (B) -0.153 (C) -0.293 (D) -0.992 (E) -1.833

3. **Multiple Choice** $\displaystyle\int \frac{dx}{(x-1)(x+3)} =$

   (A) $\dfrac{1}{4}\ln\left|\dfrac{x-1}{x+3}\right| + C$ (B) $\dfrac{1}{4}\ln\left|\dfrac{x+3}{x-1}\right| + C$

   (C) $\dfrac{1}{2}\ln|(x-1)(x+3)| + C$ (D) $\dfrac{1}{2}\ln\left|\dfrac{2x+2}{(x-1)(x+3)}\right| + C$

   (E) $\ln|(x-1)(x+3)| + C$

4. **Free Response** A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{10}\right).$$

   (a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

   (b) If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

   (c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{10}\right).$$

   Find $Y(t)$ if $Y(0) = 3$.

   (d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

Ch 6 Review Section

AP* Examination Preparation

You may use a graphing calculator to solve the following problems.

67. The spread of a rumor through a small town is modeled by $dy/dt = 1.2y(1 - y)$, where y is the proportion of the townspeople who have heard the rumor at time t in days. At time $t = 0$, ten percent of the townspeople have heard the rumor.

    (a) What proportion of the townspeople have heard the rumor when it is spreading the fastest?

    (b) Find y explicitly as a function of t.

    (c) At what time t is the rumor spreading the fastest?

68. A population P of wolves at time t years ($t \ge 0$) is increasing at a rate directly proportional to $600 - P$, where the constant of proportionality is k.

    (a) If $P(0) = 200$, find $P(t)$ in terms of t and k.

    (b) If $P(2) = 500$, find k.

    (c) Find $\lim_{t\to\infty} P(t)$.

69. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \ge 0$. After her parachute opens, her velocity satisfies the differential equation $dv/dt = -2(v + 17)$, with initial condition $v(0) = -47$.

    (a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

    (b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

    (c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?