Put answers on a separate piece of paper. Label each Section. Show all work for Free Response questions.

Quick Quiz for AP* Preparation: Sections 7.1–7.3

You may use a graphing calculator to solve the following problems.

1. **Multiple Choice** The base of a solid is the region in the first quadrant bounded by the x-axis, the graph of $y = \sin^{-1} x$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume?

   (A) 0.117 **(B)** 0.285 **(C)** 0.467 **(D)** 0.571 **(E)** 1.571

2. **Multiple Choice** Let R be the region in the first quadrant bounded by the graph of $y = 3x - x^2$ and the x-axis. A solid is generated when R is revolved about the vertical line $x = -1$. Set up, but do not evaluate, the definite integral that gives the volume of this solid.

   (A) $\displaystyle\int_0^3 2\pi(x + 1)(3x - x^2)\,dx$

   (B) $\displaystyle\int_{-1}^3 2\pi(x + 1)(3x - x^2)\,dx$

   (C) $\displaystyle\int_0^3 2\pi(x)(3x - x^2)\,dx$

   (D) $\displaystyle\int_0^3 2\pi(3x - x^2)^2\,dx$

   (E) $\displaystyle\int_0^3 (3x - x^2)\,dx$

3. **Multiple Choice** A developing country consumes oil at a rate given by $r(t) = 20e^{0.2t}$ million barrels per year, where t is time measured in years, for $0 \le t \le 10$. Which of the following expressions gives the amount of oil consumed by the country during the time interval $0 \le t \le 10$?

   (A) $r(10)$

   (B) $r(10) - r(0)$

   (C) $\displaystyle\int_0^{10} r'(t)\,dt$

   (D) $\displaystyle\int_0^{10} r(t)\,dt$

   (E) $10 \cdot r(10)$

4. **Free Response** Let R be the region bounded by the graphs of $y = \sqrt{x}$, $y = e^{-x}$, and the y-axis.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = -1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle whose diameter runs from the graph of $y = \sqrt{x}$ to the graph of $y = e^{-x}$. Find the volume of this solid.

Quick Quiz for AP* Preparation: Sections 7.4 and 7.5

You should solve the following problems without using a graphing calculator.

1. Multiple Choice The length of a curve from $x = 0$ to $x = 1$ is given by $\int_0^1 \sqrt{1 + 16x^6}\, dx$. If the curve contains the point (1, 4), which of the following could be an equation for this curve?

(A) $y = x^4 + 3$

(B) $y = x^4 + 1$

(C) $y = 1 + 16x^6$

(D) $y = \sqrt{1 + 16x^6}$

(E) $y = x + \frac{x^7}{7}$

2. Multiple Choice Which of the following gives the length of the path described by the parametric equations $x = \frac{1}{4}t^4$ and $y = t^3$, where $0 \le t \le 2$?

(A) $\int_0^2 t^6 + 9t^4\, dt$

(B) $\int_0^2 \sqrt{t^6 + 1}\, dt$

(C) $\int_0^2 \sqrt{1 + 9t^4}\, dt$

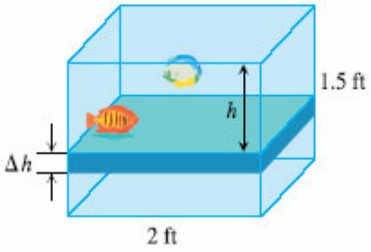
(D) $\int_0^2 \sqrt{t^6 + 9t^4}\, dt$

(E) $\int_0^2 \sqrt{t^3 + 3t^2}\, dt$

3. Multiple Choice The base of a solid is a circle of radius 2 inches. Each cross section perpendicular to a certain diameter is a square with one side lying in the circle. The volume of the solid in cubic inches is

(A) 16 **(B)** 16π **(C)** $\frac{128}{3}$ **(D)** $\frac{128\pi}{3}$ **(E)** 32π

4. Free Response The front of a fish tank is rectangular in shape and measures 2 ft wide by 1.5 ft tall. The water in the tank exerts pressure on the front of the tank. The pressure at any point on the front of the tank depends only on how far below the surface the point lies and is given by the equation $p = 62.4h$, where h is depth below the surface measured in feet and p is pressure measured in pounds/ft^2.



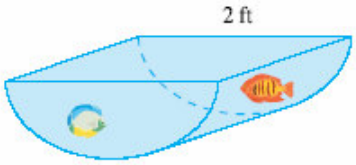
The front of the tank can be partitioned into narrow horizontal bands of height Δh. The force exerted by the water on a band at depth h_i is approximately

$$\text{pressure} \cdot \text{area} = 62.4h_i \cdot 2\Delta h.$$

(a) Write the Riemann sum that approximates the force exerted on the entire front of the tank.

(b) Use the Riemann sum from part (a) to write and evaluate a definite integral that gives the force exerted on the front of the tank. Include correct units.

(c) Find the total force exerted on the front of the tank if the front (and back) are semicircles with diameter 2 ft. Include correct units.



Ch 7 Review Section

AP* Examination Preparation

You may use a graphing calculator to solve the following problems.

53. Let R be the region in the first quadrant enclosed by the y-axis and the graphs of $y = 2 + \sin x$ and $y = \sec x$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares.

54. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(b) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(c) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

55. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{t^2 - 24t + 160}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{t^2 - 38t + 370}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour, and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, which are the hours that the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?