LESSON 1: CRITICAL VALUES

Objectives:	1. To know the meaning of critical values of a function 2. To know what is meant by absolute maxima, absolute minima, relative maxima, and relative minima 3. To locate absolute maxima and minima on a closed interval 4. To use the derivative in locating relative maxima and relative minima

Definitions

Critical Value

A **critical number** of a function *f* is a number *c* in the domain of *f* such that $f'(c) = 0$ or $f'(c)$ DNE.

Absolute Extrema

A function *f* has an **absolute maximum** (global maximum) at *c* if $f(c) \geq f(x)$ for all x in D, where *D* is the domain of *f*. The number $f(c)$ is called the **maximum value** of *f* on *D*. Similarly, *f* has an **absolute minimum** (global minimum) at *c* if $f(c) \leq f(x)$ for all *x* in *D*. The number $f(c)$ is called the **minimum value** of *f* on *D*. The maximum and minimum values are called **extreme values** of *f*.

The Extrema Value Theorem

If *f* is continuous on a closed interval [a,b], then *f* attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers *c* and *d* in [a,b].

Relative Extrema

A function has a **relative maximum** (local maximum) at c if $f(c) \geq f(x)$ when x is near c. Similarly, *f* has a **relative minimum** (local minimum) at *c* if $f(c) \leq f(x)$ when *x* is near *c*.

App. Der., Lesson 1, Cont.

Examples

1. Find all critical values

   a. $f(x) = |x|$

   b. $g(x) = x^3$

   c. $G(x) = \sqrt[3]{x^4 - 6x^2 + 8}$

   d. $f(x) = \cos^2 2x$

2. Find the absolute extrema

   a. $f(x) = x$ on $(0,1)$

   b. $g(x) = x^4 - 8x^2 + 16$ on $[-5, 0]$

Problems

1. Locate the absolute extrema for the functions given below

   a. $f(x) = x^3 - 3x^2$ on $[-1, 3]$

   b. $g(x) = 3x^{\frac{2}{3}} - 2x$ on $[-1, 1]$

   c. $h(x) = 4 - |x - 4|$ on $[1, 6]$

   d. $y = \cos \pi x$ on $[0, 2\pi]$