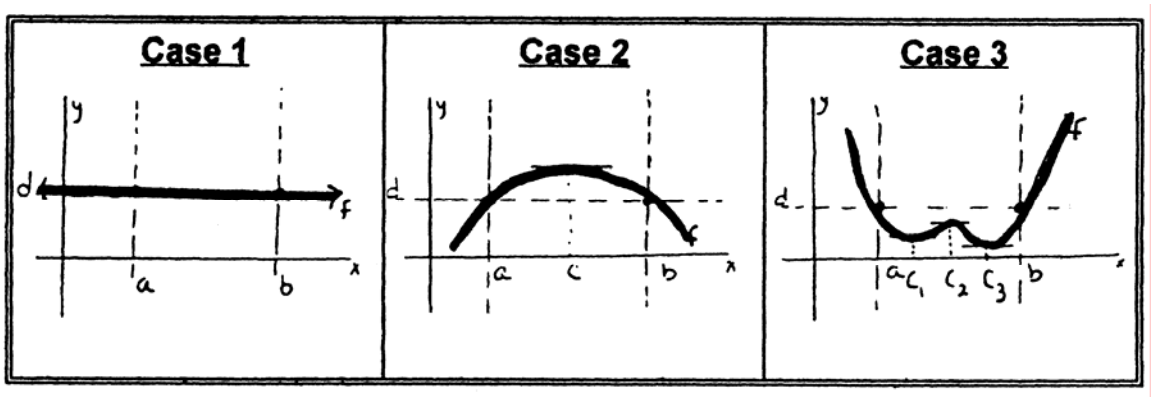
LESSON 2: THEOREMS ABOUT THE DERIVATIVE

Objectives:	1. To state and understand Rolle's Theorem 2. To state and understand the Mean Value Theorem

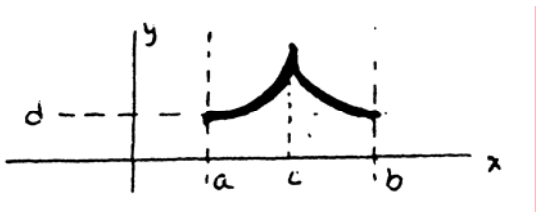
Rolle's Theorem

If f is continuous on [a,b] and differentiable on (a,b) and if f(a) = f(b), then there is at least one c in (a,b) such that $f'(c) = 0$



Corollary to Rolle's Theorem

If f is continuous on [a,b] and if f(a) = f(b), then there is at least one c in (a,b) such that f(c) is a critical value.



Mean Value Theorem

If f is continuous on [a,b] and differentiable on (a,b), then there is at least one c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Examples

1. Find all numbers *c*, that satisfy the conclusion of the Mean Value Theorem for $F(x) = 2x^3 + x^2 - x - 1$ on $[-1, 2]$.

2. Two patrol cars equipped with radar are 5 miles apart on a highway. A truck passing the first patrol car is clocked at 55 mph. Four minutes later the truck is clocked at 50 mph by the second patrol car. Prove that the truck exceeded the speed limit of 65 mph at least once.

Problems

1. Find all values *c* on (a,b) which satisfy the conclusion of the Mean Value Theorem

   a. $f(x) = \dfrac{x}{x+1}$ on $\left[-\dfrac{1}{2}, 2\right]$

   b. $g(x) = x - 2\sin x$ on $[-\pi, \pi]$

2. The height of a ball *t* seconds after it is thrown is given by $f(t) = -16t^2 + 48t + 32$

   a. Verify that f(1) = f(2)

   b. According to Rolle's Theorem, what much be the velocity at some time in the interval [1, 2]?