LESSON 3: THE FIRST AND SECOND DERIVATIVES AND CURVE SKETCHING

Objectives:	1. To use the first derivative of a function to tell where the graph is increasing or decreasing and to locate relative maxima and minima 2. To use the second derivative of a function to tell where the graph is concave up or concave down and to locate inflection points

First Derivative Test for Intervals of Increase and Decrease

Given a function f that is continuous on [a,b] and differentiable on (a,b),

a. If $f'(x) > 0$ for all $x, a < x < b,$ then f is **increasing** on (a,b).
b. If $f'(x) < 0$ for all $x, a < x < b,$ then f is **decreasing** on (a,b).
c. If $f'(x) = 0$ for all $x, a < x < b,$ then f is **constant** on (a,b).

First Derivative Test for Relative Extrema

Let c be a critical value of a continuous function f (i.e. $f'(c) = 0$ or $f'(c)$ DNE)

a. If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a **relative minimum.**
b. If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a **relative maximum.**

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MAXIMUM

MINIMUM

Applications of the Derivative, Lesson 3 cont.

Test for Concavity

If *f* is a twice-differentiable function on (a,b) and

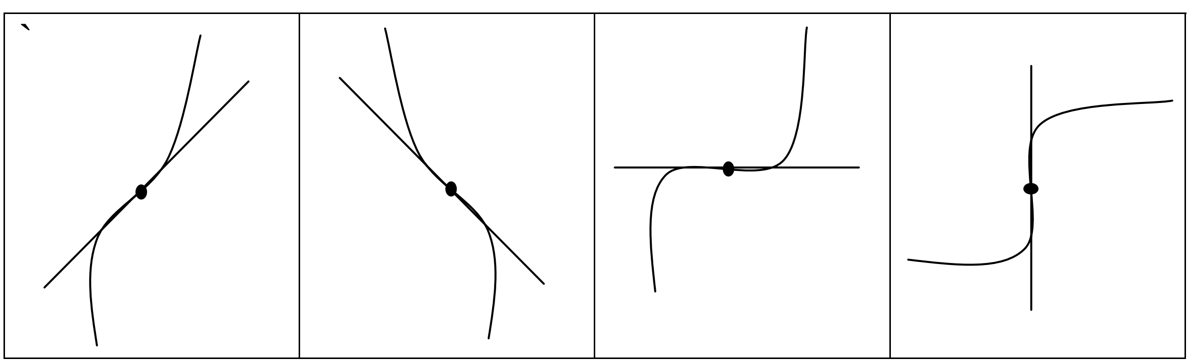
a. if $f''(x) > 0$ for all $x, a < x < b,$ then *f* is **concave up** on (a,b)
b. if $f''(x) < 0$ for all $x, a < x < b,$ then *f* is **concave down** on (a,b)

Point of Inflection

A point $\left(c, f(c)\right)$ is called an inflection point if the concavity of the graph changes at that point.

Test for Point of Inflection

If $f''(c) = 0$ or $f''(c)$ DNE, and $f''(x)$ changes from positive to negative or negative to positive at *c*, then $f(c)$ is a point of inflection.



Applications of the Derivative, Lesson 3 cont.

Examples

1. Find the intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing.

2. Find the relative extrema of $g(x) = (x^2 - 4)^{\frac{2}{3}}$

3. Discuss the curve $y = x^4 - 4x^3$ wrt solutions, maxima, minima, concavity and points of inflection. Use this information to sketch the curve.

4. For $f(x) = x^{\frac{2}{3}}(x-1)$

    a. Graph the function on a standard viewing screen on your graphing calculator and discuss its shape.

    b. Determine the intervals on which $f(x)$ is increasing or decreasing and find all relative extrema

    c. How does this information influence your opinion of the graph?

Applications of the Derivative, Lesson 3 cont.

Problems

1. Find the relative extrema of $f(x) = \frac{1}{2}x - \sin x$ on the interval $(0, 2\pi)$

2. Find the relative extrema and intervals on which $g(x) = \frac{x^4 + 1}{x^2}$ is increasing or decreasing.

3. Find the vertex of the parabola $y = -3x^2 + 9x - 7$.

4. After a drug is administered to a patient, the drug concentration in the patient's bloodstream over a two-hour period is given by

$$C = 0.29483t + 0.04253t^2 - 0.00035t^3$$

   where C is measured in milligrams and t is measured in minutes. Find the intervals on which C is increasing or decreasing.

5. Find the intervals where H(x) is ccu and ccd and find all inflection points.

$$H(x) = 6x^{\frac{1}{3}} - x^{\frac{2}{3}} - 5$$

6. Sketch $f(x) = \left|x^2 - 4\right|$.