LESSON 7: THE DIFFERENTIAL AND LINEAR APPROXIMATIONS

Objectives:	1. To define the differential dy 2. To evaluate a differential 3. To use the differential in approximations 4. To use local linearity to calculate values of a function

<u>The Differential</u>

If $y = f(x)$ is a differentiable function with the **differential** dx as the independent variable, then the **differential** dy is

$$dy = f'(x)dx$$

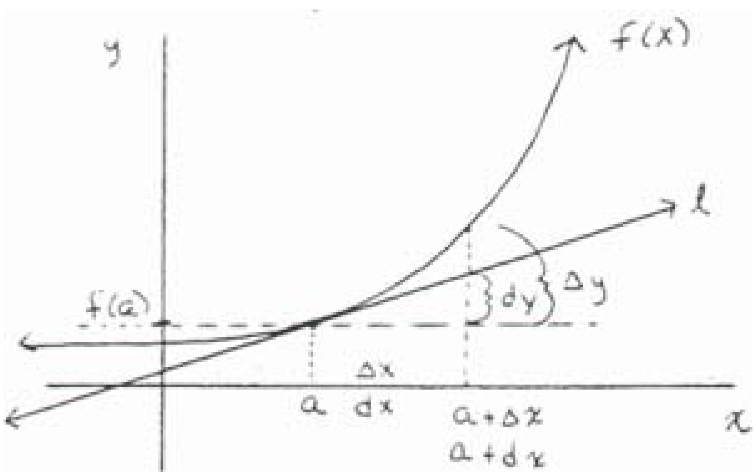
<u>Analytically</u>

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$dy = f'(x)dx$

<u>Geometrically</u>



Some exploration will show that as $\Delta x = dx$ gets smaller, the values of dy and Δy become closer to each other in value. This concept can allow the differential to be used as a tool to approximate values and in estimation of error.

<u>Examples</u>

1. Use the differential to approximate $\sqrt{66}$.

2. The measurement of the edge of a cube is 12 inches with a possible error of 0.03 inch. Use differentials to approximate the maximum possible error in computing

   a. the volume of the cube
   b. the surface area of the cube

Problems

1. Use the differential to approximate $\sqrt[3]{215}$

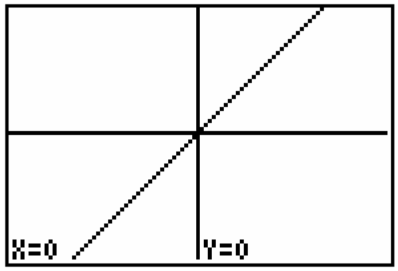
2. Find the approximate volume of a spherical shell whose inner radius is 4 inches and whose thickness is 1/16 inch.

3. For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

   a. $4+\dfrac{h}{32}$
   b. $2+\dfrac{h}{32}$
   c. $\dfrac{h}{32}$
   d. $4-\dfrac{h}{32}$
   e. $2-\dfrac{h}{32}$

Linear Approximations

The graph below appears to be linear. What linear function might it be?



Near any point the graphs of most functions look almost like a line. The Zoom-In key on the graphing calculator leads to natural conclusions about how most graphs look under a microscope. It is easy to believe that a graph represents a line when it is viewed in a small enough window. This concept is called local linearity and it can be used to produce linear approximations or a tangent line approximations for actual values on the curve.

Other uses of this "high-powered microscope" technique include investigating slope, differentiability, slope fields, and Euler's Method.

Problems

1. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

    a. Find $\frac{dy}{dx}$.

    b. Write an equation for the line tangent to the curve at the point $(4,-1)$.

    c. There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

    d. Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve.

    e. Solve the equation found in part (d) for the value of k.