LESSON 4
CROSS-SECTIONAL SLICING

Objective: To find the volume of a solid having a known integrable cross-section

Definition of Volume of a Solid

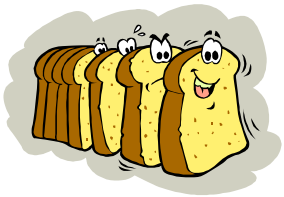
The volume of a solid of known integrable cross section area A(x), taken perpendicular to the x-axis, from x = a to x = b is the integral of A from a to b.

$$V = \int_a^b A(x)dx$$

For cross sections of area A(y), taken perpendicular to the y-axis,

$$V = \int_c^d A(y)dy$$

Procedure:
1. Sketch the solid and a typical cross section.
2. Find a formula for A(x).
3. Find the limits of integration.
4. Integrate A(x) to find the volume.



Class Activity

Divide into groups and build one of each of the following models:
1. A solid with a circular base of radius 2 inches with semicircles as cross-sections perpendicular to the base.
2. A solid with a circular base of radius 2 inches with isosceles right triangles as cross-sections perpendicular to the base.
3. A solid with a circular base of radius 2 inches with squares as cross-sections perpendicular to the base.
4. A solid with a circular base of radius 2 inches with equilateral triangles as cross-sections perpendicular to the base.

5. A solid with an elliptical base 2 inches by 4 inches with squares as cross-sections perpendicular to the base.
6. A solid with an elliptical base 2 inches by 4 inches with semicircles as cross-sections perpendicular to the base.

Examples

(l) – (6) Find the volumes of each of the solid models constructed in the Class Activity above.

7. The base of a solid is the region enclosed by an ellipse having the equation $3x^2 + y^2 = 6$. Find the volume of the solid if all plane cross-sections perpendicular to the x-axis are squares.