DERIVATIVES

LESSON 1: DEFINITION OF THE DERIVATIVE

Objectives:	1. To define the derivative of a function 2. To learn various notations for the derivative 3. To explore differentiability

Derivative of a function

The derivative of a function y = f(x) at (x, f(x)) is defined to be $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, provided the limit exists

Alternate Form of the Definition of the Derivative

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Notations for the Derivative

$$f'(x),\ \frac{dy}{dx},\ y',\ \frac{df(x)}{dx},\ D_x y$$

Differentiable Function

If $f'(x)$ exists, we say that f is **differentiable** at x. A function that is differentiable at every point of its domain is a **differentiable function.**

Examples

1. Find $f'(3)$, for $f(x) = x^2 + 4$

Examples for Lesson 1 cont.

2. Find $f'(2)$, given $f(x) = |x-2|$

3. Investigate for differentiability $y = \begin{cases} (x-1)^3 + 2 & \text{if } x < 1 \\ (x-1)^2 + 2 & \text{if } x \geq 1 \end{cases}$

4. Find all points at which the function is differentiable

$$f(x) = \frac{x^2}{x^2 - 4}$$

Problems for Lesson 1

1. Use the definition of the derivative to find $D_x g$, given that $g(x) = \sqrt{x-5}$

2. Find $f'(0)$, given $f(x) = x^{\frac{1}{3}}$

3. Find every point at which the function is differentiable.

   a. $g(x) = \sqrt{x-1}$

   b. $f(x) = \begin{cases} 4 - x^2 & \text{if } x > 0 \\ x^2 - 4 & \text{if } x \leq 0 \end{cases}$

4. $\lim_{h \to 0} \frac{|x+h| - |x|}{h}$ at $x = -2$ is

   a) -1, b) 0, c) 1, d) 2, e) DNE
   (Multiple Choice)