LESSON 10: DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Objectives: 1. To derive and use the derivatives of the exponential functions of the form e^x and a^x
2. To derive and use the derivatives of logarithmic functions of the form $\ln x$ and $\log_a x$

Recall

$$\lim_{t \to 0}(1+t)^{\frac{1}{t}} = e \approx 2.71828182845904523$$

or 2.7 A. Jackson A. Jackson Isosc. Rt. Triangle Michael Jordan

Look at the graphs of $f(x) = e^x$ and $g(x) = \ln x$.

How is it evident that they are inverses of each other?

Show algebraically that $f(x)$ and $g(x)$ are inverses of each other.

Derivations

What is $\dfrac{d}{dx} \ln x$?

What is $\dfrac{d}{dx} e^x$?

Derivatives, Lesson 10, cont.

Other related derivatives

1. $\dfrac{d}{dx}e^u = e^u \dfrac{du}{dx}$

   2. $\dfrac{d}{dx}\ln u = \dfrac{1}{u}\cdot\dfrac{du}{dx}$

3. $\dfrac{d}{dx}a^x = a^x \ln a$

   4. $\dfrac{d}{dx}a^u = a^u \ln a \dfrac{du}{dx}$

5. $\dfrac{d}{dx}\log_a x = \dfrac{1}{x \ln a}$

   6. $\dfrac{d}{dx}\log_a u = \dfrac{1}{u \ln a}\dfrac{du}{dx}$

Examples

1. Find $D_x y$, given $y = e^{-5x^3}$.

2. Find $D_x y$, given $y = e^{x \ln x^2}$.

3. Given $y = \ln(2 - 3x)^5$, find y'.

4. Given $x^2 = \ln(x^2 + y^3 - 1)$, find $D_x y$.

5. Find the equation of the line tangent to the curve $y = \ln x$ when $x = 3e$.

African
drum
music...

Derivatives, Lesson 10, Cont.

Problems

1. Find y', given

   a. $y = \ln\sqrt{x^4 - 4x}$

   b. $y = \ln(x\sqrt{x^2 - 1})$

   c. $y = \dfrac{\ln x}{x^2}$

   d. $y = \ln\sqrt{\dfrac{x+1}{x-1}}$

   e. $y = \ln\left(e^{x^2}\right)$

2. If $y = e^{ax}$, then $\dfrac{d^n y}{dx^n} =$