LESSON 4: THE CHAIN RULE

Objectives:	1. To recognize when a function is the composition of two or more functions 2. To use the chain Rule to find the derivative of a composition of functions

The Chain Rule

If $y = f(g(x))$ where $y = f(u)$ and $u = g(x)$,

then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

or

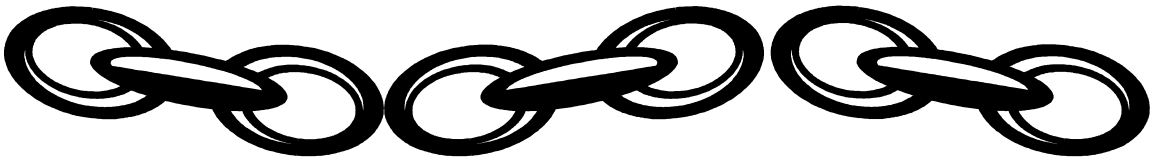
If $y = f(g(x))$, then $y' = f'(g(x))g'(x)$

Examples

1. Find the derivative of $f(x) = (2x+1)^2$

2. Find y' if $y = (x^2 + x + 1)^7$

3. Find the derivative of each of the following
   a. $f(x) = x \sin x^2$
   b. $g(x) = x \sin^2 x$
   c. $h(x) = x^2 \sin x$



Problems

1. Find the equation of the line tangent to the graph of $y = (x^2 - 5)^4 (3x - 5)^3$ at $x = 2$

2. Find $T'(w)$, given $T(w) = \left(\dfrac{5w^2 + 1}{7w^3 + 2} \right)^3$

3. Find $f'(t)$ if $f(t) = \left(\dfrac{1}{t - 3} \right)^2$

4. Show that $\dfrac{dg}{dt} = \dfrac{3t(t^2 + 3t - 2)}{(t^2 + 2t - 1)^{\frac{3}{2}}}$ for $g(t) = \dfrac{3t^2}{\sqrt{t^2 + 2t - 1}}$

5. Find $\dfrac{dy}{dx}$, given $y = \dfrac{x}{\sqrt{x^2 + 1}}$

6. For $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$, show $f'(x) = g'(x)$.

7. Show $\dfrac{d}{dx}\left[(\sin x)(\sin x + \cos x) \right] = \sin 2x + \cos 2x$