LESSON 5: PARTICLE AND LINEAR MOTION PROBLEMS

Objectives:
1. To understand that it is possible to differentiate a function more than one time
2. To obtain instantaneous velocity from the formula
3. To use the second derivative to obtain acceleration of a moving object

Higher Order Derivatives

The first derivative: Given $y = f(x)$, $\frac{dy}{dx} = y' = f'(x)$

The second derivative: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = y'' = D_x^2 y$

The nth derivative: $\frac{d^n y}{dx^n} = y^n = D_x^n y$

Velocity and Acceleration

Suppose s(t) denotes the position in feet from zero of an object in linear motion after t seconds, then

Instantaneous velocity $v(t) = s'(t)$ ft/sec

Acceleration $a(t) = v'(t) = s''(t)$ ft/sec^2

Examples

1. (1998 AB 14 MC)
   A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

   A) 1 B) 2 C) 3 D) 4 E) 5

2. (1993 AB 26 MC)
   A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

   A) -5.19 B) 0.74 C) 1.32 D) 2.55 E) 8.13

3. (1983 AB FR 1)
   A particle moves along the x-axis so that at time *t* its position is given by $x(t) = t^3 - 6t^2 + 9t + 11$

   a. What is the velocity of the particle at $t = 0$?

   b. During what time intervals is the particle moving to the left?

   c. What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

Problems

1. (1979 AB 4)

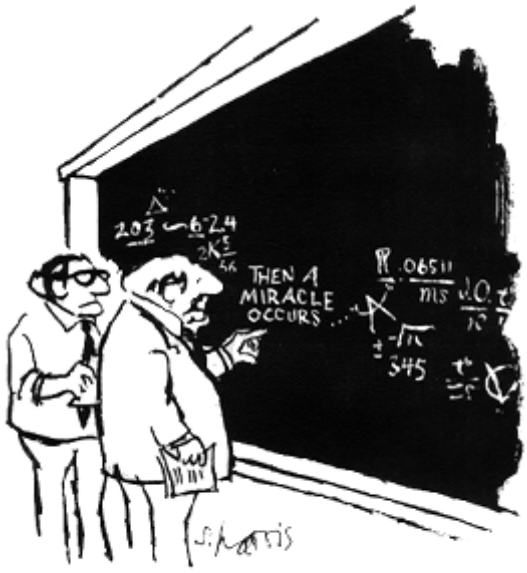
   A particle moves along a line so that at any time *t* its position is given by $x(t) = 2\pi t + \cos 2\pi t$

   a. Find the velocity at time *t*.
   b. Find the acceleration at time *t*.
   c. What are all values of *t*, $0 \le t \le 3$, for which the particle is at rest?

2. (1999 AB 1)

   A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \ge 0$.

   a. In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
   b. Find the acceleration of the particle at time *t* = 1.5. Is the velocity of the particle increasing at *t* –1.5? Why or why not?



"I think you should be more explicit here in step two."