

LESSON 6: IMPLICIT DIFFERENTIATION

Objectives	<ol style="list-style-type: none">1. To understand what it means to say that a function is represented explicitly or implicitly2. To find the derivative of a function that is defined implicitly
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Implicit and Explicit Functions

Implicit Form

Explicit Form

Derivative

$$xy = 1$$

$$y = \frac{1}{x}$$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

Guidelines for Implicit Differentiation

1. Differentiate both sides of the equation with respect to x .
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx .

Examples

1. Explore the derivatives of:
 - a. $y = 3x + 5$ and $3x - y = -5$
 - b. $y = \sqrt{1 - x^2}$ and $x^2 + y^2 = 1$
2. Find $\frac{dy}{dx} \Big|_{(0,-2)}$ for $x^3 - xy + y^2 = 4$

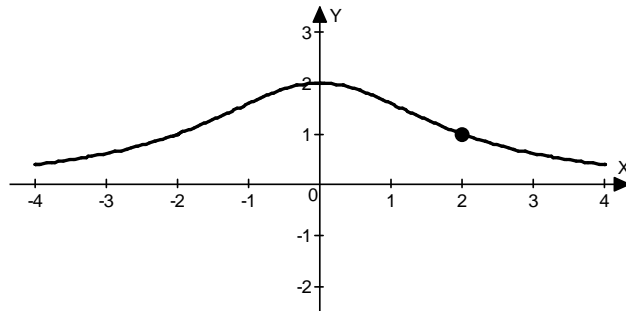
Problems

1. Find $D_x y$, given $x^2 + y^3 = 8x^3 y^2$
2. Find y' for the function defined implicitly by
 $2x^3 y^5 - 3x^2 y^4 + 5y^2 - x^3 y - 7 = 0$
3. Find $\frac{dy}{dx}$ for $\tan(x + y) = x$
4. Determine the equation of the tangent line to the graph at the indicated point

a. Witch of Agnesi

$$(x^2 + 4)y = 8$$

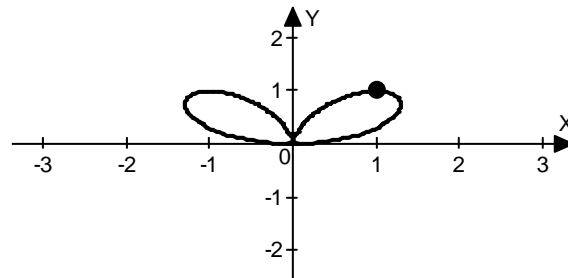
Point (2,1)



b. Bifolium

$$(x^2 + y^2)^2 = 4x^2 y$$

Point (1,1)



5. Sketch the intersecting graphs of the given equations and show that they are orthogonal. (Two graphs are **orthogonal** if at their point(s) of intersection, their tangent lines are perpendicular to each other.)

$$2x^2 + y^2 = 6$$

$$y^2 = 4x$$