

LESSON 7: DERIVATIVES OF INVERSE FUNCTIONS

Objective: 1. To find the derivative of a function when it is defined implicitly

Inverse Function

A function f has an inverse, denoted f^{-1} if

1. f is 1-1 on its domain
2. f is strictly increasing
3. f is strictly decreasing

If $f(x)$ and $g(x)$ are inverses, then $f(g(x)) = g(f(x)) = x$

Derivative of an Inverse Function

Defined explicitly

Write as $y = f^{-1}(x)$ and apply differentiation rules

Defined implicitly

$y = f(x)$ Original Function

$x = f(y)$ Inverse

$1 = f'(y) \frac{dy}{dx}$ Implicit Differentiation

$\frac{dy}{dx} = \frac{1}{f'(y)}$ Derivative of the Inverse

Examples

1. Find $D_x f^{-1}(2)$, given $f(x) = x^3 + 10$

2. Find $D_x f^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, given $f(x) = \cos x, 0 \leq x \leq \pi$

Problem

1. Find $D_x f^{-1}\left(-\frac{1}{3}\right)$, given $f(x) = \frac{x}{x^2 - 4}$ on $(-2, 2)$