

# INTEGRATION

## LESSON 1: ANTIDIFFERENTIATION RULES AND TECHNIQUES

- Objectives:
1. To understand that antidifferentiation is the inverse operation to differentiation
  2. To state and use the notation and rules of antidifferentiation
  3. To learn and use the technique of substitution

### Antiderivative

A function  $F$  is called an antiderivative of a function  $f$  if for every  $x$  in  $f$ ,  $F'(x) = f(x)$ .

### Notation for the Antiderivative (The Indefinite Integral)

$$\int f(x)dx = F(x) + C, \text{ where } C \text{ is a constant}$$

### Example

1. Find the antiderivatives of  $1, x, x^2, x^3, x^4, x^n$ .

### Properties and Rules for Integration

$$\int cf(x)dx = c \int f(x)dx$$
$$\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

## Examples

1.  $\int (x^3 - 3x) dx$
2.  $D_x \int (3x^2 + 5x + 2) dx$

## Trig Properties

$$\frac{d}{dx}(\sin x) = \cos x \rightarrow \int \cos x dx = \sin x + C$$

$$\frac{d}{dx}(\cos x) = -\sin x \rightarrow \int \sin x dx = -\cos x + C$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \rightarrow \int \sec^2 x dx = \tan x + C$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x \rightarrow \int \csc^2 x dx = -\cot x + C$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \rightarrow \int \sec x \tan x dx = \sec x + C$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x \rightarrow \int \csc x \cot x dx = -\csc x + C$$

## Examples

1.  $\int (\theta + 3\sec^2 \theta) d\theta$
2.  $\int \frac{t^2 + 2}{t^2} dt$

## Rectilinear Motion

If  $a(t)$ =acceleration, then

$$v(t) = \int a(t)dt + v_0$$

$$s(t) = \int v(t)dt + s_0$$

## Examples

1. If a baseball were thrown straight up at a speed of 90 mph (132 ft/sec), how high would the ball go?
2. What are the equations of motion ( $s(t)$ ,  $v(t)$ ,  $a(t)$ ) for King Kong's 1350 foot fall off the Empire State Building?
3. The acceleration of a particle moving along a straight line is given for any time  $t$  by  $a(t) = 6t^2 + 4t - 3$ . If the velocity is 3 when  $t=0$ , what is the velocity when  $t=2$ ?

## **Substitution Method**

Let  $f$  and  $g$  be functions such that  $f$  and  $g'$  are continuous functions, then  $\int f(g(x)) \cdot g'(x) dx$  can be evaluated by following the steps below:

1. Substitute  $u = g(x)$  and  $du = g'(x) dx$  to obtain the integral

$$\int f(u) du$$

2. Integrate wrt  $u$
3. Replace  $u$  by  $g(x)$  in the result

## **Examples**

1.  $\int (x^2 - x + 5)^8 (2x - 1) dx$
2.  $\int \sin 5x dx$
3.  $\int 4x^2 \cos x^3 dx$
4.  $\int 3 \tan^3 \theta \sec^2 \theta d\theta$

## **Problems**

1.  $\int (5z + 8)^{\frac{1}{3}} dz$
2.  $\int x \sin x^2 dx$
3.  $\int \frac{x+1}{(x^2 + 2x - 3)^2} dx$
4.  $\int 2\pi y (8 - y^{\frac{3}{2}}) dy$