INTEGRATION

LESSON 1: ANTIDIFFERENTIATION RULES AND TECHNIQUES

Objectives:	1. To understand that antidifferentiation is the inverse operation to differentiation 2. To state and use the notation and rules of antidifferentiation 3. To learn and use the technique of substitution

Antiderivative

A function F is called an antiderivative of a function f if for every x in f, $F'(x) = f(x)$.

Notation for the Antiderivative (The Indefinite Integral)

$$\int f(x)dx = F(x) + C, \text{ where } C \text{ is a constant}$$

Example

1. Find the antiderivatives of $1, x, x^2, x^3, x^4, x^n$.

Properties and Rules for Integration

$$\int cf(x)dx = c\int f(x)dx$$

$$\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

Examples

1. $\int (x^3 - 3x)dx$
2. $D_x \int (3x^2 + 5x + 2)dx$

Trig Properties

$$\frac{d}{dx}(\sin x) = \cos x \rightarrow \int \cos x dx = \sin x + C$$
$$\frac{d}{dx}(\cos x) = -\sin x \rightarrow \int \sin x dx = -\cos x + C$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \rightarrow \int \sec^2 x dx = \tan x + C$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x \rightarrow \int \csc^2 x dx = -\cot x + C$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x \rightarrow \int \sec x \tan x dx = \sec x + C$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x \rightarrow \int \csc x \cot x dx = -\csc x + C$$

Examples

1. $\int (\theta + 3\sec^2 \theta)d\theta$
2. $\int \frac{t^2 + 2}{t^2} dt$

Rectilinear Motion

If a(t)=acceleration, then

$$v(t) = \int a(t)dt + v_0$$

$$s(t) = \int v(t)dt + s_0$$

Examples

1. If a baseball were thrown straight up at a speed of 90 mph (132 ft/sec), how high would the ball go?

2. What are the equations of motion (s(t), v(t), a(t)) for King Kong's 1350 foot fall off the Empire State Building?

3. The acceleration of a particle moving along a straight line is given for any time t by $a(t) = 6t^2 + 4t - 3$. If the velocity is 3 when t=0, what is the velocity when t=2?

Substitution Method

Let f and g be functions such that f and g' are continuous functions, then $\int f(g(x)) \cdot g'(x)dx$ can be evaluated by following the steps below:

1. Substitute $u = g(x)$ and $du = g'(x)dx$ to obtain the integral
$$\int f(u)du$$
2. Integrate wrt u
3. Replace u by g(x) in the result

Examples

1. $\int (x^2 - x + 5)^8 (2x - 1)dx$
2. $\int \sin 5x dx$
3. $\int 4x^2 \cos x^3 dx$
4. $\int 3\tan^3 \theta \sec^2 \theta d\theta$

Problems

1. $\int (5z + 8)^{\frac{1}{3}} dz$
2. $\int x \sin x^2 dx$
3. $\int \frac{x+1}{(x^2 + 2x - 3)^2} dx$
4. $\int 2\pi y(8 - y^{\frac{3}{2}})dy$