LESSON 3: THE DEFINITE INTEGRAL

Objectives:	1. To define and geometrically describe a Riemann Sum 2. To develop and use the definite integral to solve the problem of area 3. To state and apply the Fundamental Theorem of Calculus

Riemann Sum

If f is a function defined on [a,b], and [a,b] is partitioned arbitrarily

$$a = x_0 < x_1 < x_2 < ... < x_{k-1} < x_k = b$$

where Δx_j is the width of the j^{th} subinterval and c_j is any point in the j^{th} subinterval, then

$$\sum_{j=1}^{k} f(c_j)\Delta x_j$$

is called a Riemann sum of f.

Example

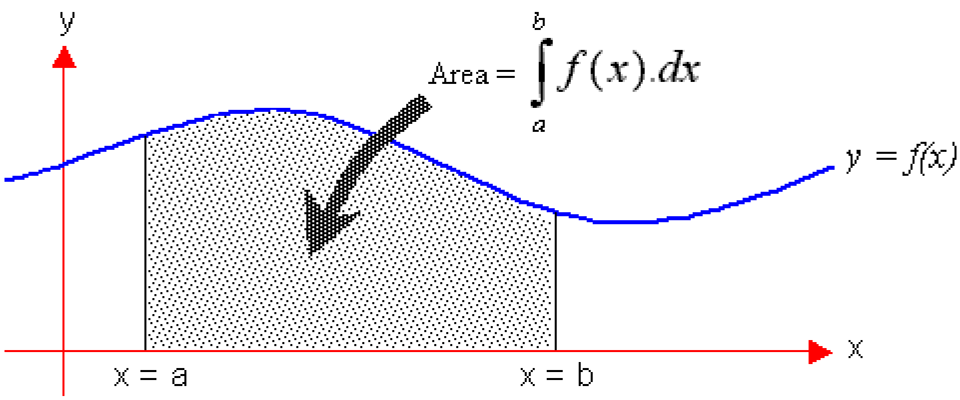
1. Find the Riemann sum for $f(x) = x^2$ on $[0, 2]$

   Partition P: 0, 0.5, 0.75, 1.1, 1.4, 1.8, and 2

   $c_1 = 0.25, c_2 = 0.5, c_3 = 1, c_4 = 1.2, c_5 = 1.5, c_6 = 2$

The Area Function

Derivation: For y = f(x), a continuous, positive function on [a,b], define A(x) to be the area under the graph from a to b.



Examples

1. Find the area under the curve of $y = x^2$ on [0,1]
2. Find the area under the curve of $y = x^2$ on [1,2]

Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_a^x f(t)dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

If f is continuous at every point of [a,b], and if F is any antiderivative of f on [a,b], then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Problems

1. $\int_0^2 (2x+5)dx$
2. Find the area under the graph of $f(x) = \sin x$ over the interval $\left[\frac{\pi}{4}, \frac{5\pi}{6}\right]$.
3. Find the area under the graph of $f(x) = |x-2|$ over the interval [0,6].
4. $\int_{-4}^{4} \left(4 - |x|\right) dx$
5. $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$