

LESSON 4: PROPERTIES OF THE DEFINITE INTEGRAL

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| Objectives: | <ol style="list-style-type: none">1. To state and apply properties of the definite integral2. To state and apply the Mean Value Theorem for integrals3. To state and apply the Average Value of a Function on an Interval |
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Properties of the Definite Integral

If f is continuous on $[a,b]$, then f is integrable on $[a,b]$

$$\int_a^b kf(x)dx = k \int_a^b f(x)dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

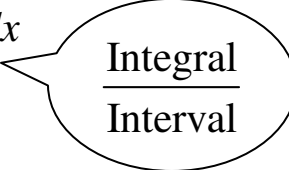
$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Mean Value Theorem for Integrals

If f is continuous on $[a,b]$, then there exists a value “ c ” on (a,b)

such that $\int_a^b f(x)dx = f(c)(b-a)$.

Average Value of a Function on an Interval

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$


Integral
Interval

Examples

1. Write $\int_{-2}^1 f(x) dx + \int_5^{-2} f(x) dx + \int_1^5 f(x) dx$ as a single integral

2. $\int_{-3}^4 |x^2 - 4| dx$

3. $D_x \int_2^x t^3 dt$

4. $D_x \int_2^{x^2} t^3 dt$

5. $D_x \int_x^2 t^{-1} \cos^3 t^2 dt$

Problems

1. $\int_1^4 \frac{u-1}{\sqrt{u}} du$

2. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 4 \sec \theta \tan \theta d\theta$

3. $\int_0^4 |x^2 - 4x + 3| dx$

4. $D_x \int_{x^2}^2 t^{-1} \cos^3 t^2 dt$

5. Find the value of c guaranteed by the Mean Value Theorem for $f(x) = x^3$ on $[0,2]$.

6. Find the average value on the interval $[-2,2]$ of $f(x) = 4 - x^2$.