

## LESSON 3: PROPERTIES OF LIMITS

Objectives: 1. To state and use properties of limits  
2. To derive and use the Squeeze Property

### Properties of Limits

Constant Multiplier Property       $\lim_{x \rightarrow b} (k \cdot f(x)) = k \cdot \lim_{x \rightarrow b} f(x)$

Sum /Diff. Property       $\lim_{x \rightarrow b} [f(x) \pm g(x)] = \lim_{x \rightarrow b} f(x) \pm \lim_{x \rightarrow b} g(x)$

Product Property       $\lim_{x \rightarrow b} [f(x) \cdot g(x)] = \lim_{x \rightarrow b} f(x) \cdot \lim_{x \rightarrow b} g(x)$

Quotient Property       $\lim_{x \rightarrow b} \left[ \frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow b} f(x)}{\lim_{x \rightarrow b} g(x)}$

Power/Root Property       $\lim_{x \rightarrow b} [f(x)]^n = \left[ \lim_{x \rightarrow b} f(x) \right]^n$

Squeeze Property      If  $a(x) \leq f(x) \leq b(x)$  and  
 $\lim_{x \rightarrow c} a(x) = \lim_{x \rightarrow c} b(x) = L$   
then  $\lim_{x \rightarrow c} f(x) = L$

## Examples

1.  $\lim_{x \rightarrow -2} \sqrt{\frac{x^3 + 2x^2 - 5x - 1}{x + 6}}$

2. Use the Squeeze Property to show that  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

3.  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 6x}$

4. Show  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

## Problems

1.  $\lim_{x \rightarrow 0} \frac{x + 1 - \cos x}{x}$

5.  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x}$

2.  $\lim_{x \rightarrow 0} \frac{\sin^2 5x}{x^2}$

6.  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$

3.  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

7.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\cot x}$

4.  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan 3x}{\tan 5x}$

8. Use the Squeeze Property to

show that  $\lim_{x \rightarrow 0} x \cdot \sin\left(\frac{1}{x}\right) = 0$