

LESSONS 4/5

ONE-SIDED LIMITS

LIMITS THAT FAIL TO EXIST

- Objectives:
1. To recognize when a limit does not exist
 2. To use one-sided limits
 3. To recognize and evaluate infinite limits

One-sided Limits

Right-hand: $\lim_{x \rightarrow c^+} f(x)$ The limit of f as x approaches c from the right

Left-hand: $\lim_{x \rightarrow c^-} f(x)$ The limit of f as x approaches c from the left

Two-sided Limits

A function $f(x)$ has a limit as x approaches c iff the right-hand and left-hand limits at c exist and are equal.

$$\lim_{x \rightarrow c} f(x) \text{ exists iff } \lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x)$$

Examples

1. a. $\lim_{x \rightarrow 0^+} \frac{|x|}{x}$

b. $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$

c. $\lim_{x \rightarrow 0} \frac{|x|}{x}$

2. a. $\lim_{x \rightarrow 0} \frac{1}{x}$

b. $\lim_{x \rightarrow 0} \frac{1}{x^2}$

3. $\lim_{x \rightarrow 2} \frac{|x^2 - 4|}{x - 2}$

4. $\lim_{x \rightarrow 1} \frac{2x}{x-1}$

5. $\lim_{x \rightarrow 1^+} \frac{x+2}{1-x}$

6. (Calculator Exploration) $\lim_{x \rightarrow 0} \left(\sin \frac{1}{x} \right)$

Problems

1. Given the greatest integer function $y = [x]$, find

a. $\lim_{x \rightarrow 1^+} [x]$

b. $\lim_{x \rightarrow 1^-} [x]$

c. $\lim_{x \rightarrow -1^+} [x]$

d. $\lim_{x \rightarrow -1^-} [x]$

2. Let $f(x) = \begin{cases} 1-x & \text{if } x < -1 \\ 1 & \text{if } x = -1 \\ x^3 & \text{if } x > -1 \end{cases}$, find

a. $\lim_{x \rightarrow -1^-} f(x)$

b. $\lim_{x \rightarrow -1^+} f(x)$

c. $\lim_{x \rightarrow -1} f(x)$

3. a. $\lim_{x \rightarrow 3^-} \frac{1}{(x^2 - 9)}$

b. $\lim_{x \rightarrow 3^+} \frac{1}{(x^2 - 9)}$

c. $\lim_{x \rightarrow 3} \frac{1}{(x^2 - 9)}$

4. $\lim_{x \rightarrow 3^-} \frac{4x^3}{9 - x^2}$

5. $\lim_{x \rightarrow 5^+} \frac{x - 5}{x^2 - 25}$

6. $\lim_{x \rightarrow 1^+} \frac{x^2 + x + 1}{x^3 - 1}$

7. $\lim_{x \rightarrow 0^+} \left[\frac{1}{x} - \frac{1}{x^2} \right]$