

## LESSON 6: LIMITS TO INFINITY

- Objectives: 1. To learn the meaning of  $\lim_{x \rightarrow \pm\infty} f(x)$
2. To learn techniques for evaluating limits to infinity

### Rules for Limits to Infinity for a Rational Function

- a. If  $N < D$ , then  $\lim_{x \rightarrow \pm\infty} f(x) = 0$
- b. If  $N = D$ , then  $\lim_{x \rightarrow \pm\infty} f(x) = \text{Ratio } \frac{N}{D}$
- c. If  $N > D$ , then  $\lim_{x \rightarrow \pm\infty} f(x) = \pm\infty$

### Examples

1. a.  $\lim_{x \rightarrow \infty} \frac{1}{x}$                       b.  $\lim_{x \rightarrow -\infty} \frac{1}{x}$
2. a.  $\lim_{x \rightarrow \infty} \frac{x}{2x-1}$                       b.  $\lim_{x \rightarrow -\infty} \frac{x}{2x-1}$
3. a.  $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+2}}{4x-1}$                       b.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+2}}{4x-1}$
4.  $\lim_{x \rightarrow \infty} (\sqrt{x+4} - \sqrt{x})$

Problems for Lesson 6

1. (Squeeze Property)  $\lim_{x \rightarrow \infty} \frac{3\cos^2 x}{x}$

2.  $\lim_{x \rightarrow \infty} (\sin x)$

3.  $\lim_{x \rightarrow \infty} (x \ln x)$

4.  $\lim_{x \rightarrow \infty} \frac{x^2 + 3}{x^4 - 2x}$

5.  $\lim_{x \rightarrow \infty} \frac{x^4 - 2x}{x^2 + 3}$

6. a.  $\lim_{x \rightarrow \infty} \frac{\sqrt{5x^2 + 2}}{4x - 1}$

b.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{5x^2 + 2}}{4x - 1}$

7.  $\lim_{x \rightarrow \infty} \frac{8t^3 + 4t - 5}{3t^4 - 5t^2 + 7}$

8.  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 + 5x - 7} - x \right)$

9.  $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 - x}}$

10.  $\lim_{x \rightarrow \infty} \frac{1}{2x + \sin x}$