

LESSON 7: CONTINUITY

- Objectives:
1. To recognize that $f(x)$ is continuous at $x=b$ iff
$$\lim_{x \rightarrow b} f(x) = f(b)$$
 2. To locate points of discontinuity

Continuity

A function $f(x)$ is continuous at a point $(b, f(b))$ if

$$\lim_{x \rightarrow b^+} f(x) = \lim_{x \rightarrow b^-} f(x) = f(b)$$

Types of Discontinuities

1. Point – *Removable*
2. Jump
3. Infinite

Examples

Investigate for Continuity

1. $f(x) = \frac{2x}{x-1}$ at $x = 2$, at $x = 1$

2. $f(x) = \begin{cases} x+1 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

3. $g(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Problems for Lesson 7, Continuity

Investigate for Continuity

1. $f(x) = \frac{2x - 2}{x^2 + x - 2}$

2. $f(x) = \frac{x^3 - x}{x^3 - 4x}$

3. $h(x) = \frac{|x| - 2}{x - 2}$

4. $f(x) = \begin{cases} -1 & \text{if } x \leq -2 \\ x^2 - 5 & \text{if } |x| < 2 \\ 3x - 6 & \text{if } x \geq 2 \end{cases}$

5. $f(t) = \frac{t}{t^2 + 1}$

6. This is a good one!

$$g(x) = \begin{cases} \csc\left(\frac{\pi x}{6}\right) & \text{if } |x - 3| \leq 2 \\ 2 & \text{if } |x - 3| > 2 \end{cases}$$