LESSON 8: APPLICATIONS OF LIMITS

Objectives:
1. To use the limit concept in solving the problem of tangents
2. To develop the limit definition of the derivative
3. To apply the concept of limit to average velocity and instantaneous velocity

Slope of the Tangent Line to the Graph of y = f(x) at (a, f(a))

$$m_{\tan} = \lim_{h \to 0} \left(m_{\sec} \right) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Equation of Line Tangent to the Graph of y = f(x) at (a, f(a))

$$y - f(a) = m_{\tan}(x - a)$$

Average Rate of Change

The **average rate of change** of a quantity over a period of time is the amount of change divided by the time it takes.

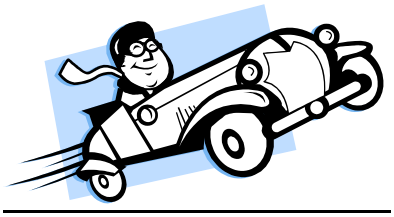
i.e. $\frac{f(b) - f(a)}{b - a}$ for (a, f(a)) and (b, f(b))

For example:

If $f(1) = 2$ and $f(3) = 7$, then the average rate of change, written $\frac{\Delta y}{\Delta x}$,

would be $\frac{\Delta y}{\Delta x} = \frac{f(3) - f(1)}{3 - 1} = \frac{7 - 2}{3 - 1} = \frac{5}{2}$

Lesson 8 cont.



Rectilinear Motion

For two moments in time, t and t+h, and for the position equation y= s(t), then **average velocity**

$$v_{ave} = \frac{\text{distance traveled}}{\text{change in time}} = \frac{s(t+h)-s(t)}{(t+h)-t} = \frac{s(t+h)-s(t)}{h}$$

and **instantaneous velocity**

$$v = \lim_{h\to 0}\left(v_{ave}\right) = \lim_{h\to 0}\frac{s(t+h)-s(t)}{h}$$

Examples for Lesson 8

1. Find the equation of the line tangent to $f(x) = 3x^2 + 5x - 2$ at x = 1.
2. Find the equation of the line tangent to the graph of $g(x) = 4x + 3$ when $x = 3$.
3. An object moves along a straight line with equation of motion $s(t) = 2t^2 - 8t$. Find the average speed of this object over the time intervals [5, 6], [5, 5.5], [5, 5.1]. Also, find the instantaneous velocity at t = 5.

Problems for Lesson 8

1. Find the slope of the tangent line for
   a. $y = x^2 + 1$ at $(2,5)$
   b. $y = -2x^2 + 3x + 1$ at $(0,1)$
   c. $f(x) = \frac{1}{x}$ at $(4, \frac{1}{4})$

2. Find the equation of the line tangent to the graph of $h(x) = 2x^3 + 8$ at $x = 2$

3. The motion of an object is given by $s(t) = -4.9t^2 + 19.6t$ where t is in seconds and s is in meters. Find v at t = 1, 2, and 3 seconds. Can you describe the motion of the object from this information?