

## LESSON 3: INTEGRATION OF POWERS OF SINE AND COSINE

- Objectives:
1. To see that all integrals involving powers of sin and cos can be classified into three cases
  2. To integrate powers of sin and cos

### Integrating Powers of Sine and Cosine

**Case I:**  $\int \sin^n ax \cos^m ax dx$  where at least one of  $n$  and  $m$  are positive, odd integers  
Use  $\sin^2 ax + \cos^2 ax = 1$

**Case II:**  $\int \sin^n ax \cos^m ax dx$  where both  $n$  and  $m$  are positive, even integers  
Use  $\sin^2 ax = \frac{1 - \cos 2ax}{2}$  or  $\cos^2 ax = \frac{1 + \cos 2ax}{2}$

**Case III:**  $\int \sin ax \cos b x dx, \int \cos ax \cos b x dx, \int \sin ax \sin b x dx$   
Use  $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$   
or  $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

### Examples

1.  $\int \cos^5 x \sin^2 x dx$
2.  $\int \cos^2 x \sin^4 x dx$
3.  $\int \cos 3x \sin 4x dx$

### Problems

1.  $\int \cos^3 2x dx$
2.  $\int \sin^2 x \cos^2 x dx$
3.  $\int \sin x \sin 3x dx$
4.  $\int \cos^3 x \sin x dx$