LESSON 5: EXPONENTIAL GROWTH AND DECAY

Objectives:	1. To derive and apply the formula for growth and decay 2. To derive and apply Newton's Law of Cooling 3. To apply the logistic growth model

Law of Exponential Change

An amount of a quantity increases or decreases at a rate proportional to the amount present. If the amount present at time $t = 0$ is $y = y_0$, then the differential equation

$\frac{dy}{dt} = ky$ becomes $y = y_0 e^{kt}$.

Newton's Law of Cooling

The rate at which an object's temperature is changing at any given time is proportional to the difference between its temperature and the temperature of the surrounding medium. If the temperature at t=0 is T_0, then the differential equation

$\frac{dT}{dt} = -k(T - T_s)$ becomes $T - T_s = (T_0 - T_s)e^{-kt}$.

App. Der., Lesson 5, cont.

Logistic Growth Model

If population growth has a maximum population M, called the **carrying capacity**, and if we assume the relative growth rate is proportion to $1-\frac{P}{M}$, then this relationship in differential form is

$$\frac{\frac{dP}{dt}}{P}=k\left(1-\frac{P}{M}\right) \text{ or } \frac{dP}{dt}=\frac{k}{M}P(M-P).$$

The solution of this general logistic differential equation is

$$P=\frac{M}{1+Ae^{-kt}}.$$

M = maximum population or carrying capacity, k = constant of proportionality, P = population, t = time, A = a constant determined by the appropriate initial condition.

Examples

1. World Population Growth: If the population of the world in 1950 was 2520 million and in 1980 was 4450 million,
   a. Find the relative growth rate per year
   b. Develop a growth model
   c. Predict the world population in the year 2000
   d. Predict the world population in the year 3000

App. Der., Lesson 5, cont.

2. The half-life of radium-226 is 1590 years. A sample of radium-226 has a mass of 100 mg.
   a. Find a formula for the mass that remains after t years.
   b. Find the mass after 1000 years correct to the nearest milligram.
   c. When will the mass be reduced to 30 mg?

3. The logistic model can be used to illustrate the spread of a rumor. A small town has 1000 inhabitants. At 8:00 a.m., 80 people have heard a rumor. By noon half the town has heard it. At what time will 90% of the population have heard the rumor?