

LESSON 6: SLOPEFIELDS AND EULER'S METHOD

Objective:

To understand and solve differential equations using Slopefields and Euler's Method - extensions of local Linearity

Slopefields

Some of the “new topics” in the 1997-8 course description can also be taught as extensions of local linearity. Slopefields and Euler's Method are used to understand and solve equations involving derivatives (i.e., differential equations) and are best understood as extensions of local linearity.

Both the AB and the BC course description include

- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth.

The BC course description includes the following:

- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and derivatives of implicitly defined functions
- Numerical solution of differential equations using Euler's Method

Euler's Method

Euler's Method is a way to build a piecewise linear function that approximates the solution to a differential equation. Given an initial value and the derivative function, the method draws a tangent line to the curve at the initial point using the derivative, and then moves along the tangent line to a new x . A new slope is calculated using the derivative function and a new tangent line is drawn and we again move along the tangent line to the next x . For concave down curves, the approximation will be too high since the tangent lines lie above the curve.

Slopefields are used to understand differential equations in a geometrical representation. Euler's Method allows us to investigate the differential equation in an numerical representation.

If a function is defined implicitly, it may not be possible to solve the equation for the y -value with a given x -value or to graph the function in a $y=$ menu. However, using slopefields it is possible to see a picture of the function and use Euler's Method to approximate for a particular value for y .

WORKSHEET FOR EULER'S METHOD

differential equation:

initial condition:

Step	x	approximate y	$\Delta y = (\text{slope}) \Delta x$	actual y
1				
2				
3				
4				
5				

