Teacher Notes

Appendix A

Introduction to Graphing Piecewise (Split-Defined) Functions

Objective

- Graph portions of functions restricted by explicit domains

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 30 minutes

Abstract

This is an introduction to graphing piecewise functions using the **TEST Menu** on the graphing handheld in conjunction with its function graphing abilities.

Management Tips and Hints

Prerequisites

Students should:

- know how to produce graphs.
- be able to set appropriate windows manually and use the zoom functions to adjust windows (standard, trig, decimal).
- understand the nature of compound inequalities and the meaning of the Boolean operators "or" and "and."

This activity should be done prior to the limits activity. It could also be used in class prior to calculus in which piecewise-defined functions are first introduced.

Student Engagement

The questions are useful in determining whether the student can verbalize what is being done as well as push buttons.

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Evidence of Learning

Students will be able to:

- produce a correct graph.
- describe the input necessary for someone else to produce an appropriate graph.

Common Student Errors/Misconceptions

Students who graph in connected mode using multiplication may incorrectly assume that the "jump lines" are part of the graph.

Extensions

The following example can be used to check understanding.

$$f(x) = \begin{cases} -x - 5, x < -3 \\ -x^2, -2 \le x < 2 \\ x + 3, x \ge 2 \end{cases}$$

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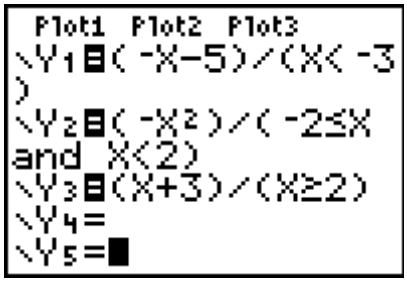
Activity Solutions

1. $(-x - 5)/(x < -3)$

2. $(-x^2)/(-2 \le x \text{ and } x < 2)$

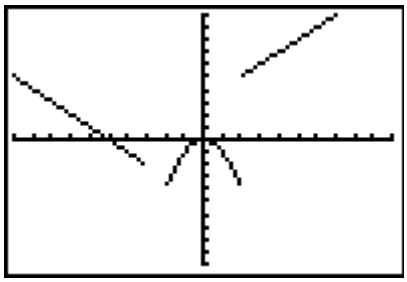
3. $(x + 3)/(x \ge 2)$

Answers to **1**–**3** in **Y=** editor:



4. The standard viewing window is reasonable. All branches exist in that window, and all the restrictions are contained between $-10 \le x \le 10$.

5.



6. $f(-6) = 1$, $f(-1) = -1$, $f(0) = 0$, $f(2) = 5$, $f(9) = 12$, $f(12) = 15$

Because you can use the table or the **Yvalue** function from the **VARS Menu** or just manually plug in the values, the answers to how students arrived at the values will vary. However, if a student is using the **value** feature of the **CALC Menu** and the standard viewing window, $f(12)$ will appear as "INVALID." This is a good checkpoint to see whether the student understands that the x-values in the window do actually restrict what the graphing handheld can show you.

7. $f(-2.6)$ is undefined because no branch of the function is defined between -3 and -2. Any method the student uses to evaluate the function at -2.6 should produce that result.

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EXPLORATIONS ™

Appendix A

Objective

- Graph portions of functions restricted by explicit domains

Materials

- TI-84 Plus / TI-83 Plus

Introduction to Graphing Piecewise (Split-Defined) Functions

Introduction

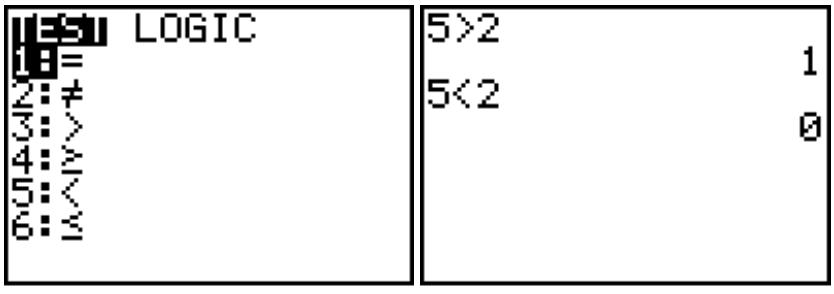
In many branches of mathematics, you are asked to work with functions that have various split-defined branches. This activity will show you how to produce those functions on the graphing handheld screen.

Exploration

On the graphing handheld, the **TEST Menu** compares values in the traditional way.

The results of this menu are simple:

- A value of 1 is produced if your relationship is true.
- A value of 0 is produced if your relationship is false.



You can use these operators to restrict the portions of a graph that you see on screen.

For example, look at $f(x) = x^2$, $x \geq -2$.

- In the **Y=** editor, input **$(x^2)/(x \geq -2)$**.
- Select **6:Standard** from the **ZOOM Menu** to see the graph in the standard viewing window.

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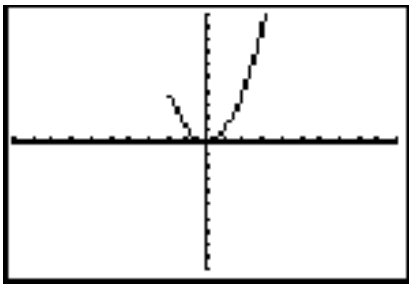
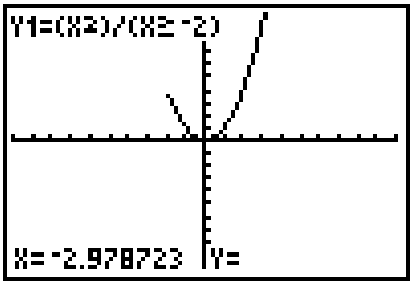
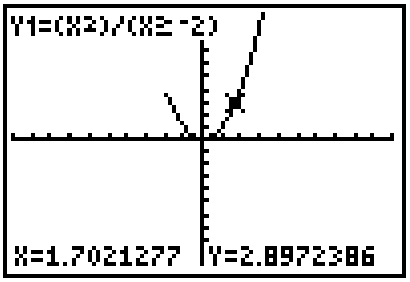
Your screen should look like the one shown.



See what happens when you **TRACE** the function.

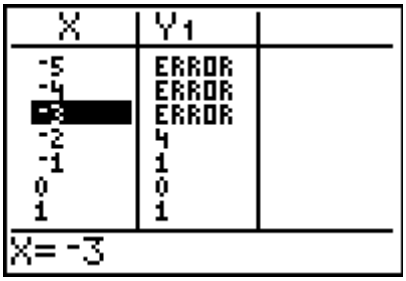
- When evaluating a value less than or equal to −2, a division by 0 situation occurs, thus leaving part of the graph undefined and not graphed.
- Notice that the *y*-value is blank and that there is no cursor on the screen.



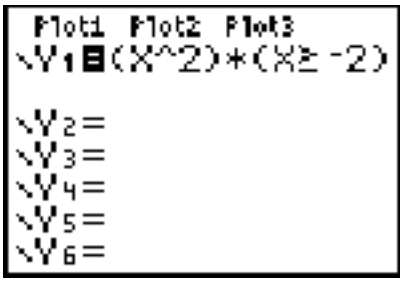


Now look at the **TABLE**.

In the table, the "missing" part of the graph shows as an error.



You can also use multiplication to define a piecewise function.



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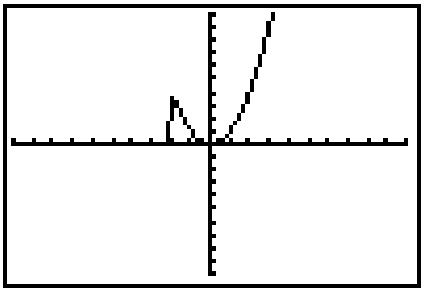
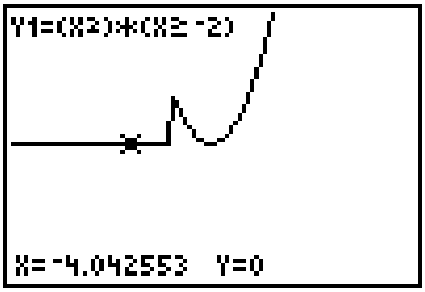
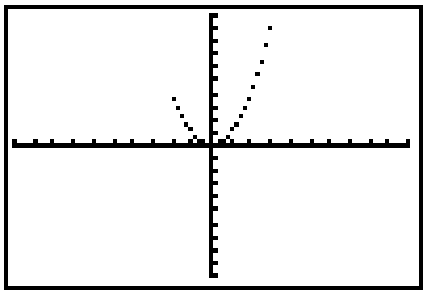
In **Connected** plotting mode, this method can produce unwanted *jump lines*. The graphing handheld is simply *connecting* the last point it drew to the next point it draws. If instructed to multiply, it will produce values of zero when the Boolean expression $x \leq -2$ is false.

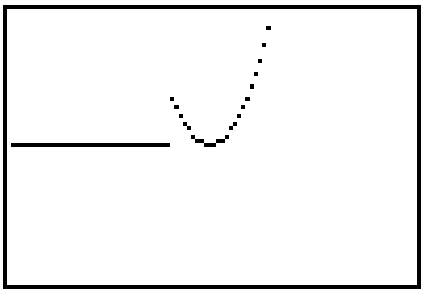


Turn the axes off to get a better look.



In **Dot** plotting mode, the graphing handheld will not show the "jump lines," but it also may not show as many points as you would like it to show. With the axes off, it will show the line $y = 0$.





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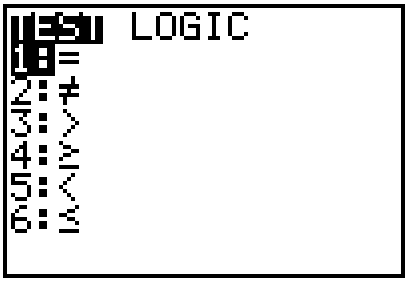
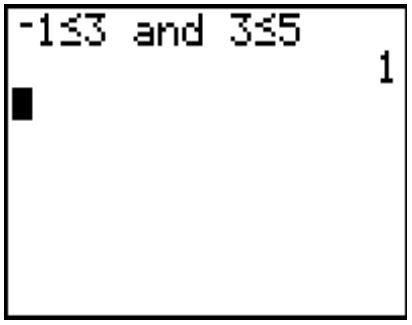
You can find the compound inequalities *and* and *or* under the **TEST LOGIC Menu** on the graphing handheld.



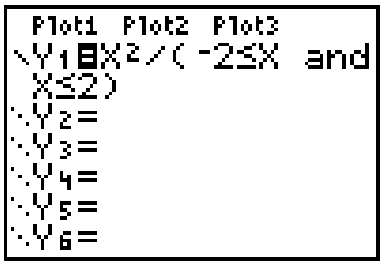
The graphing handheld tests these relationships the same way you would do if you were mentally comparing the values. When you input the branches connected with the Boolean piece, the graphing handheld will produce a result of either 1 or 0, depending on whether the expression is true or false.

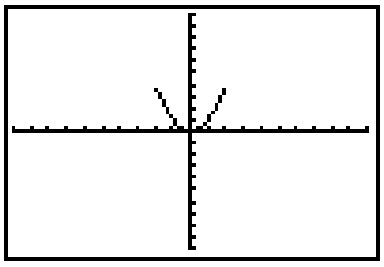
For example, test $-1 \le 3 \le 5$.

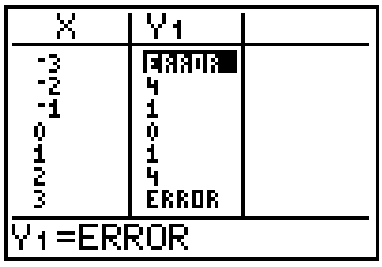


These Boolean operators can be used to evaluate and graph $f(x) = x^2$, $-2 \le x \le 2$.

Enter **$(x^2)/(-2 \le x$ and $x \le 2)$** in the **Y=** editor.







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View the table, and notice the "error" in the restricted branches.

Although there are many ways to produce a full piecewise function, the best visual representation is produced by entering each branch as a function in the **Y=** editor.

For example, to graph

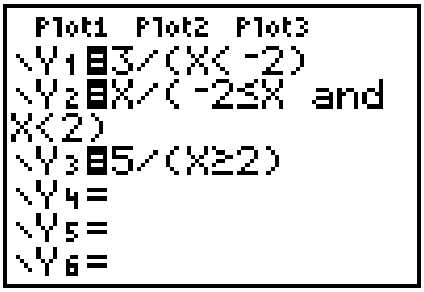
$$f(x) = \begin{cases} 3, x < -2 \\ x, -2 \le x < 2 \\ 5, x \ge 2 \end{cases}$$

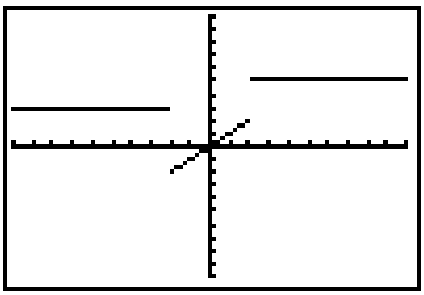
you should

- Enter the first branch in **Y1**.
- Enter the second branch in **Y2**.
- Enter the third branch in **Y3**.

Graph the function in an appropriate viewing window.

You should see screens similar to the screenshots shown.





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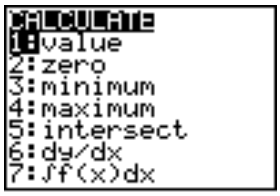
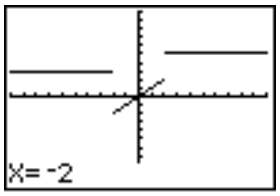
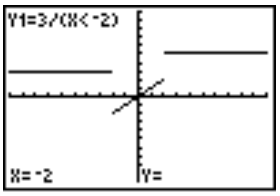
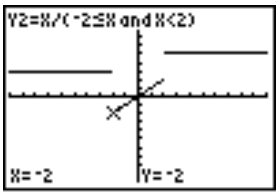
When you trace, you will have to use the up and down cursor arrows to move between branches of the function. You can investigate the values of the branches by using **1:VALUE**, found in the **CALC Menu**, and jumping between branches. This is a very effective way to examine the behavior at the transition points. For example, if you press [CALC] [VALUE] and enter ⁻2 for X, you will see that *y* has a proper value in **Y2** but is undefined in **Y1**.









You can also use the **TABLE** to view the transitions.

X	Y1	Y2
-4	3	ERROR
-3	3	ERROR
-2	ERROR	-2
-1	ERROR	-1
0	ERROR	0
1	ERROR	1
2	ERROR	ERROR

Y1=ERROR

X	Y2	Y3
-4	ERROR	ERROR
-3	ERROR	ERROR
-2	-2	ERROR
-1	-1	ERROR
0	0	ERROR
1	1	ERROR
2	ERROR	5

Y3=ERROR

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Graph this piecewise function:

$$f(x) = \begin{cases} -x - 5, x < -3 \\ -x^2, -2 \le x < 2 \\ x + 3, x \ge 2 \end{cases}$$

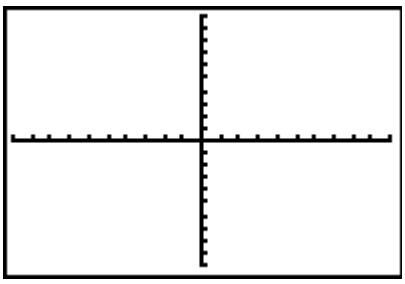
1. Write your input for **Y1**.

2. Write your input for **Y2**.

3. Write your input for **Y3**.

4. What is a reasonable window to use to produce the graph? Why?

5. Transfer the graph you see to one set of the included axes. Be sure to label the ends of the axes according to the window you chose.



6. Find the following function values $f(-6)$, $f(-1)$, $f(0)$, $f(2)$, $f(9)$, $f(12)$. Explain how you found the values.

7. Does the function have a value for $f(-2.6)$? Explain why or why not.

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