

LESSON 5: FUNCTIONS AND GRAPHS

- Objectives:
1. To learn what is meant by the terms function, domain, and range
 2. To use interval notation when describing domain and range
 3. To perform composition of functions
 4. To graph piecewise functions
 5. To study symmetry properties

Important Terms

Function, domain, range, symmetry, even and odd functions, absolute value

Composition of Functions

Given two functions f and g , $(f \circ g)(x) = f(g(x))$

$$D_{f \circ g} = \{x : x \in D_g \text{ and } g(x) \in D_f\}$$

Symmetry

x-axis, y-axis, origin, even and odd functions

Example

1. Graph the piecewise function

$$f(x) = \begin{cases} 1 + \sqrt{-7 - x} & \text{for } x \leq -7 \\ \frac{(9 - x^2)}{50} & \text{for } -7 < x < -1 \\ \sin(x) & \text{for } -1 \leq x \leq 3 \\ \frac{1}{(x - 3)} & \text{for } x > 3 \end{cases}$$

Problems

1. Given $f(x) = 4x^2 - 3x + 5$, evaluate and simplify the following $f(-2)$, $f(3)$, $f(b)$, $f(x^3)$, $f(x-2)$, $f(-2)$, $f(a+h) - f(a)$
2. If $f(x) = x - 1$ and $g(x) = 1 + x^2$, for what values of x is $f(g(x)) = g(f(x))$?
3. If the solutions of the quadratic function $f(x) = 0$ are -1 and 2 , then the solutions of $f\left(\frac{x}{2}\right) = 0$ are what?
4. Graph the piecewise function

$$g(x) = \begin{cases} 2.4x + 33.8 & \text{for } x < -12 \\ \sqrt{(169 - x^2)} & \text{for } -12 \leq x < 0 \\ 13 - x & \text{for } 0 \leq x < 10 \\ -\frac{1}{4}x^2 + 5x - 19 & \text{for } x \geq 10 \end{cases}$$

5. Determine the symmetries of the following equations
 - a. $f(x) = x(4 - x^2)$
 - b. $x^2 + 4y^2 = 4$
 - c. $y = \frac{1}{x}$
6. Graph each function and state its domain and range
 - a. $f(x) = x^{\frac{2}{5}}$
 - b. $f(x) = (x - 3)^{\frac{2}{3}}$