LESSON 8: INVERSES AND LOGARITHMS

Objectives:
1. To find the inverse of a function geometrically and algebraically
2. To define the logarithm function
3. To review the properties of log functions

Inverse of a function

1. Interchange x and y
2. Resolve for y

Notation

$y = f^{-1}(x)$

Lipstick test

1. (1,0)
2. $y = x^2$
3. $y = x^3$
4. $y = 10^x$
5. $y = e^x$

Logarithmic Function

$y = \log_a x$ is the inverse of $y = a^x$ $(a < 0, a = 1)$

Properties of Logarithms

For any real numbers $a > 1$, $x > 0$, and $y > 0$

1. $\log_a 1 = 0$
2. $\log_a a^x = x$
3. $a^{\log_a x} = x$
4. $\log_a (xy) = \log_a x + \log_a y$
5. $\log_a \frac{x}{y} = \log_a x - \log_a y$
6. $\log_a x^y = y \log_a x$
7. $\log_a x = \frac{(\log_b x)}{\log_b y}$

Examples

1. Find the inverse function and its domain for $f(x)=\dfrac{3x-2}{4x+1}$.

2. Show that $g(x)=\dfrac{-7x-5}{4x-3}$ is an inverse function to $f(x)=\dfrac{3x-5}{4x+7}$.

Problems

1. Find the inverse function and its domain for each of the following functions (if the inverse exists).
   a. $f(x)=2x^2+2, \quad x\le -1$
   b. $g(x)=7x-5$
   c. $h(x)=\dfrac{-x-2}{5x-1}$

2. Suppose that f is a function that is defined for all real numbers. Which one of the following conditions assures that f has an inverse?
   a. The function *f* is periodic
   b. The graph of *f* is symmetric wrt the y-axis
   c. The graph of *f* is concave up
   d. The function *f* is strictly increasing
   e. The function *f* is continuous

3. Solve
   a. $2^x=16$
   b. $3^x=729$
   c. $3^{(2-8x)}=27^{(3x+1)}$
   d. $\log_4 x=1$
   e. $\log_4 x+\log_4(x+6)=2$

4. Find the following logs
   a. $\log_2(32)$
   b. $\log_2\dfrac{1}{8}$
   c. $\log_2 4^5$