DATE ____________________

3.1–3.3 Concepts Worksheet

NAME ____________________

Differentiation

1. Given the following information about differentiable functions $f(x)$ and $g(x)$ at $x = 2$ and $x = 3$,

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	1/3	-3
3	3	-4	2π	5

determine the value of:

__________ a) $\frac{d}{dx}\{2f(x)\}$ at $x = 2$

__________ b) $\frac{d}{dx}\{f(x) + g(x)\}$ at $x = 3$

__________ c) $\frac{d}{dx}\{f(x) \cdot g(x)\}$ at $x = 3$

__________ d) $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\}$ at $x = 2$

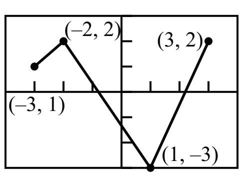
__________ e) $\frac{d}{dx}\{f(g(x))\}$ at $x = 2$

__________ f) $\frac{d}{dx}\{\sqrt{f(x)}\}$ at $x = 2$

__________ g) $\frac{d}{dx}\left\{\frac{1}{g(x)}\right\}$ at $x = 3$

__________ h) If $h(x) = \sqrt{f^2(x) + g^2(x)}$, then find $h'(2)$.

2.



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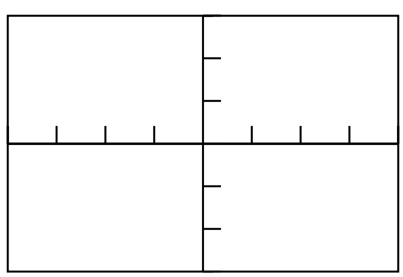


The graph of $f(x)$ with domain $[-3, 3]$ is composed of line segments as shown above.

(a) Sketch the graph of $f'(x)$ on the grid above.

(b) Name the x-coordinate of each point of discontinuity of $f'(x)$ over $(-3, 3)$.

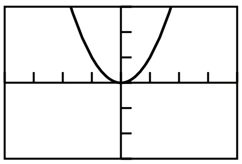
____

Continued

Concept Connectors

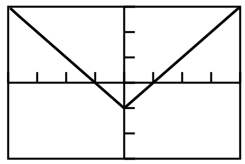
3. What points do you suspect of being points of discontinuity of the derivatives of these graphs? (Give the x-coordinates of the points of discontinuity of $f'(x)$.)

(a) $f(x) = x^2$



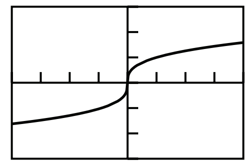
[–4, 4] by [–3, 3]

(b) $f(x) = |x| - 1$



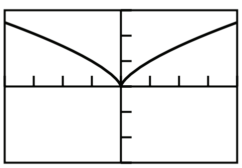
[–4, 4] by [–3, 3]

(c) $f(x) = x^{1/3}$



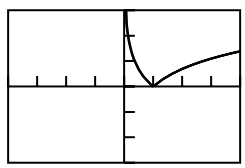
[–4, 4] by [–3, 3]

(d) $f(x) = x^{2/3}$



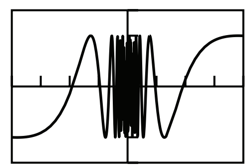
[–4, 4] by [–3, 3]

(e) $f(x) = |\ln x|$



[–4, 4] by [–3, 3]

(f) $f(x) = 2 \sin \frac{6}{x}$



[–4, 4] by [–3, 3]

4. You may not have *formally* arrived at the "suspect" points called for above. Formal limit computations as described in Appendix A3 would rigorously prove derivative discontinuities. However, some of the examples used above would still seem difficult. In general, which characteristics on a curve would make you believe that the slope of a tangent line to the curve at that point is nonexistent?

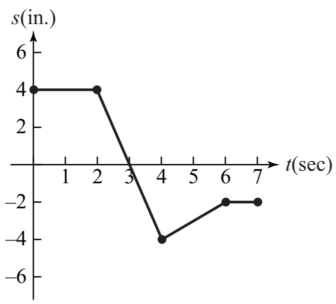
DATE ____________________

3.4 Concepts Worksheet

NAME ____________________

Velocity, Speed, and Acceleration

1. The graph shows the position $s(t)$ of a particle moving along a horizontal coordinate axis.

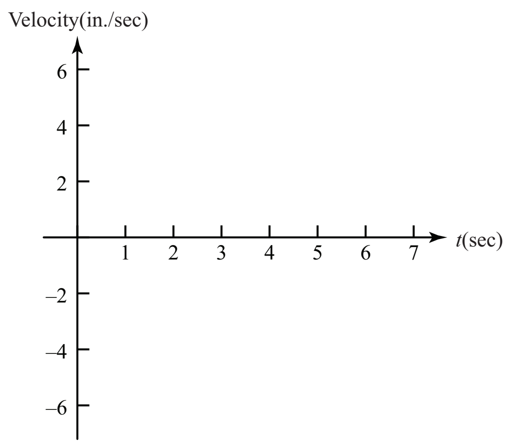


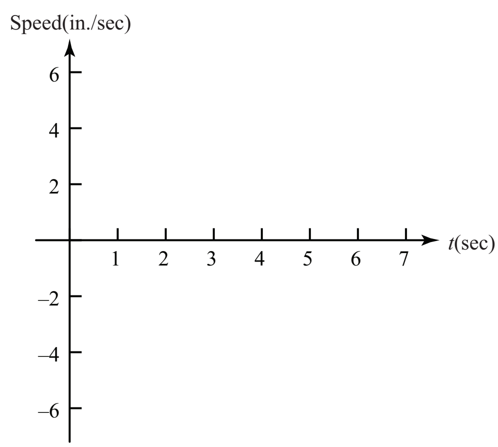
(a) When is the particle moving to the left? ____________________

(b) When is the particle moving to the right? ____________________

(c) When is the particle standing still? ____________________

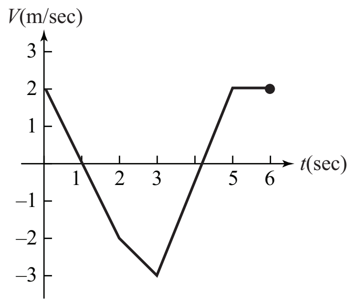
(d) Graph the particle's velocity and speed (where defined).





(e) When is the particle moving fastest? ____________________

2. The graph shows the velocity $v = f(t)$ of a particle moving along a horizontal coordinate axis.

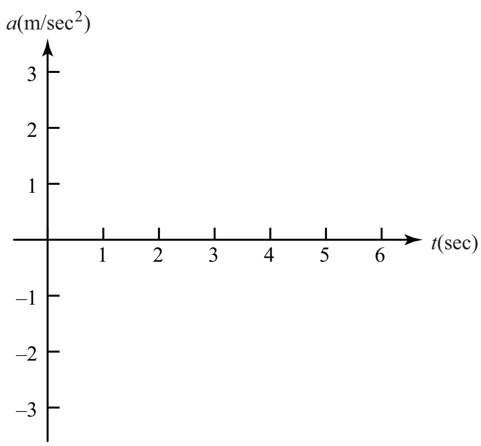


(a) When does the particle reverse direction? ____________________

(b) When is the particle moving at a constant speed? ______________

(c) When is the particle moving at its greatest speed? ______________

(d) Graph the acceleration (where defined).



3. A particle moves along a vertical coordinate axis so that its position at any time $t \geq 0$ is given by the function $s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 4$, where s is measured in centimeters and t is measured in seconds.

(a) Find the displacement during the first 6 seconds.

____

(b) Find the average velocity during the first 6 seconds.

____

(c) Find expressions for the velocity and acceleration at time t.
$v(t) =$ ________________ $a(t) =$ ________________

(d) For what values of t is the particle moving downward?

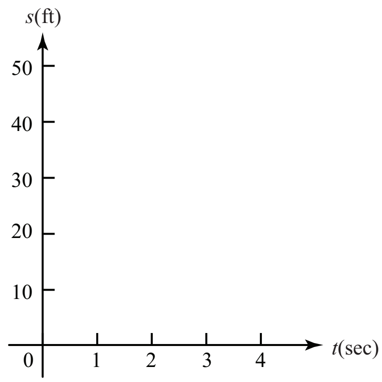
____

Continued

4. The values of the coordinate s of a moving body for various values of t are given below.

t(sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
s(ft)	40.0	35.0	30.2	36.0	48.2	45.0	38.2	16.0	0.2

(a) Plot s versus t, and sketch a smooth curve through the given points.



(b) Estimate the velocity at each of the following times.
At $t = 0.5$ sec, $v \approx$ ________________.
At $t = 2.5$ sec, $v \approx$ ________________.
At $t = 3$ sec, $v \approx$ ________________.

(c) At what approximate values of t does the particle change direction?

____

(d) At what approximate value of t is the particle moving at the greatest speed?

____

