DATE

4.2 Concepts Worksheet

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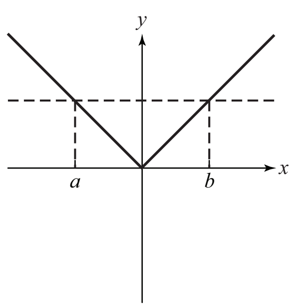
Theorems of Calculus

Rolle's Theorem states: If a function is continuous at every point on a closed interval $[a, b]$ and differentiable on every point of its interior (a, b) and $f(a) = f(b) = 0$, then there is at least one number c between a and b at which $f'(c) = 0$.

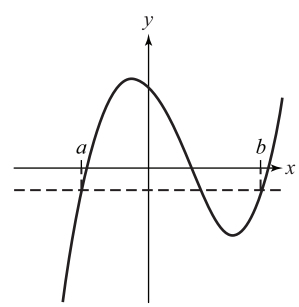
A variation of Rolle's Theorem includes broader conditions: If function $f(x)$ is continuous at every point in a closed interval $[a, b]$ and $f(a) = f(b)$, then there exists at least one critical point of $f(x)$ between $x = a$ and $x = b$.

1. Using this variation of Rolle's Theorem, find and mark the critical points on the following graphs, if applicable. If not applicable, explain why not.

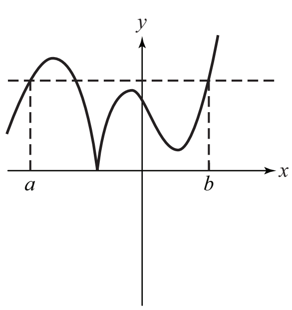
(a)



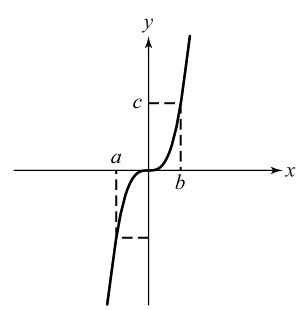
(b)



(c)



(d)



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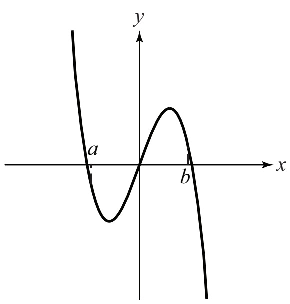
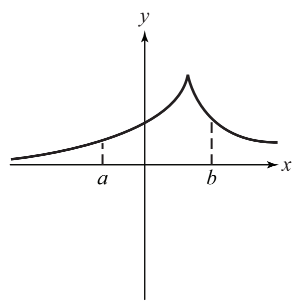
Continued

2. Given the functions below as drawn over the interval $[a, b]$, are the conditions of the Mean Value Theorem met? (If not, why not?) If conditions are met, locate the value(s) of c that satisfy the equation $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. Draw the parallel tangent lines and secant line implied in the Mean Value Theorem.

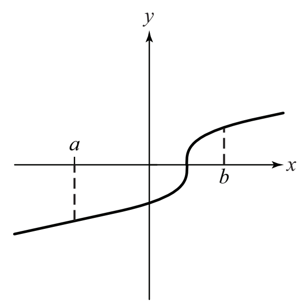
(a)



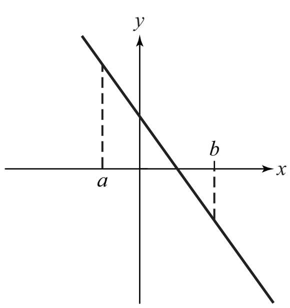
(b)



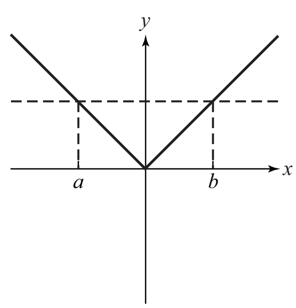
(c)



(d)



(e)



4.2 Concepts Worksheet

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Concept Connectors

3. Suppose that $f(x)$ is a function with continuous first and second derivatives on the closed interval $[1, 3]$ whose values for f and f' at $x = 1$ and $x = 3$ are given below:

x	$f(x)$	$f'(x)$
1	5	2
3	7	-1

(a) Prove there exists a value of c, $1 < c < 3$, such that $f'(c) = 1$.

(b) Prove there exists a value of d, $1 < d < 3$, such that $f''(d) = -\frac{3}{2}$.

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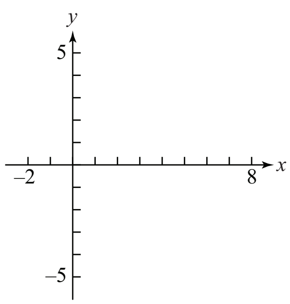
4.3 Concepts Worksheet

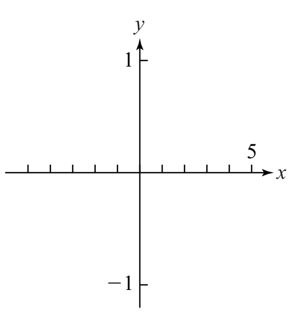
Graph Sketching Using Derivatives

1. Sketch a graph of a differentiable function $f(x)$ over the closed interval $[-2, 7]$, where $f(-2) = f(7) = -3$ and $f(4) = 3$. The roots of $f(x) = 0$ occur at $x = 0$ and $x = 6$, and $f(x)$ has properties indicated in the table below:

x	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$4 < x < 7$
$f'(x)$	positive	0	positive	1	positive	0	negative
$f''(x)$	negative	0	positive	0	negative	0	negative



2. Sketch a graph of the continuous even function $g(x)$ over the closed interval of x values $[-5, 5]$ having a range of $g(x)$ values $[-1, 0]$. For $x \geq 0$, roots of $g(x) = 0$ occur at every whole number k and roots of $g'(x) = 0$ occur at $\frac{k}{2}$. The first and second derivatives of $g(x)$ exist everywhere except at $x = k$. Furthermore, $g''(x) > 0$ for every $x \neq k$.



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3. Sketch a function $h(x)$ from the following information:

(a) $h(-x) = -h(x)$

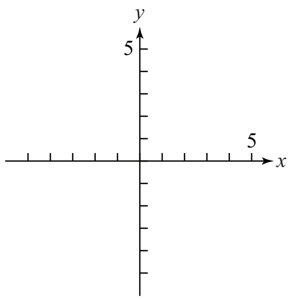
(b) $\lim_{x \to 0^+} h(x) = \infty$

(c) $\lim_{x \to +\infty} h(x) = 0$

(d) For $x > 0$, $h(x) = 0$ only at $x = 1$

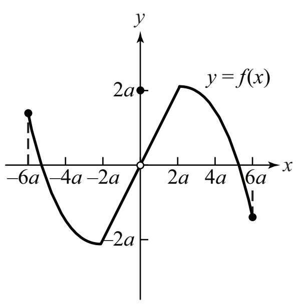
(e) For $x > 0$, $h'(x) = 0$ only at $x = 2$

(f) For $x > 0$, $h''(x) = 0$ only at $x = 3$



Concept Connectors

4. The graph of $f(x)$ is shown on the closed interval $[-6a, 6a]$:



Answer the following questions regarding $f(x)$:

(a) For $x \neq 0$, the graph of $f(x)$ has symmetry about the _______________, that is $f(-x) =$ _______________.

(b) f has point(s) of discontinuity at $x =$ _______________.

(c) $\lim_{x \to 0} f(x) =$ _______________.

(d) The zeros of $f(x)$ occur at $x =$ _______________.

(e) $f'(x)$ does not exist at $x =$ _______________.

(f) $f''(x) < 0$ for the x interval(s) _______________.