AP Calculus
Chapter 6 Review
(No Integration by Parts)

Name: KEY
Block: ______

For questions 1 – 26, integrate each of the following indefinite integrals.

1. $\int \frac{3x}{\sqrt[3]{x^2+3}}\,dx$

$\boxed{\frac{9}{4}(x^2+3)^{2/3}+C}$

2. $\int \sec x\,dx$

$\boxed{\ln|\sec x+\tan x|+C}$

3. $\int \frac{x-1}{x+1}\,dx$

$\boxed{x-2\ln|x+1|+C}$

4. $\int \sin x\,dx$

$\boxed{-\cos x+C}$

5. $\int b^{4x}\,dx$
~~where b is a constant~~

$\boxed{\frac{1}{4\ln b}\cdot b^{4x}+C}$

6. $\int \tan x\,dx$

$\boxed{-\ln|\cos x|+C}$

7. $\int \cot^2 x\,dx$

$\boxed{-\cot x-x+C}$

8. $\int \frac{(x+1)^2}{x^{1/3}}\,dx$

$\boxed{\frac{3}{8}x^{8/3}+\frac{6}{5}x^{5/3}+\frac{3}{2}x^{2/3}+C}$

9. $\int \cos^2 x\,dx$

$\boxed{\frac{\sin(2x)}{4}+\frac{x}{2}+C}$

10. $\int \frac{dx}{4+9x^2}$

$\boxed{\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)+C}$

11. $\int \cos t\,dt$

$\boxed{\sin(t)+C}$

~~12. $\int \sin^{-1} x\,dx$~~

SKIP... Requires "Parts"

13. $\int \sin^2 x\,dx$

$\boxed{\frac{x}{2}-\frac{\sin(2x)}{4}+C}$

14. $\int \frac{dx}{x\sqrt{x^2-4}}$

$\boxed{\frac{1}{2}\sec^{-1}\left|\frac{x}{2}\right|+C}$

15. $\int \cot x\,dx$

$\boxed{\ln|\sin x|+C}$

16. $\int \csc x\,dx$

$\boxed{-\ln|\csc x+\cot x|+C}$

17. $\int \sqrt{x}(3-4x)\,dx$

$\boxed{2x^{3/2}-\frac{8}{5}x^{5/2}+C}$

18. $\int \frac{dx}{\sqrt{-x^2-2x}}$

$\boxed{\sin^{-1}(x+1)^2+C}$

19. $\int \tan^2 x\,dx$

$\boxed{\tan x-x+C}$

20. $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$\boxed{\sin^{-1}(e^x)+C}$

21. $\int \csc^2 x\,dx$

$\boxed{-\cot(x)+C}$

22. $\int xe^{-x^2+3}\,dx$

$\boxed{-\frac{1}{2}e^{-x^2+3}+C}$

23. $\int \sec^2 x\,dx$

$\boxed{\tan x+C}$

24. $\int \frac{3+x}{x^2+1}\,dx$

$\boxed{3\tan^{-1}(x)+\frac{1}{2}\ln(x^2+1)+C}$

25. $\int (\cos t-\sin t)^2\,dt$

$\boxed{t-\frac{\sin(2t)}{2}+C}$

26. DERIVE (SHOW EVERY STEP) $y=y_0e^{kt}$ from $\frac{dy}{dt}=ky$ and $y(0)=y_0$. [Should be on your notecards!]

See other pages

27. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

a) Find the slope of the graph of f at the point where $x = 1$. $\boxed{1/2}$

b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$ $\boxed{4.1}$

c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

$\boxed{y = \pm\sqrt{x^3 + x + 14}}$

d) Use your solution from part c to find $f(1.2)$ $\boxed{\approx 4.114}$

28. If $\dfrac{dy}{dx} = y\sec^2 x\, dx$ and $y = 5$ when $x = 0$, then $y =$

A $e^{\tan x} + 4$

B $e^{\tan x} + 5$

$\boxed{\text{C} \quad 5e^{\tan x}}$

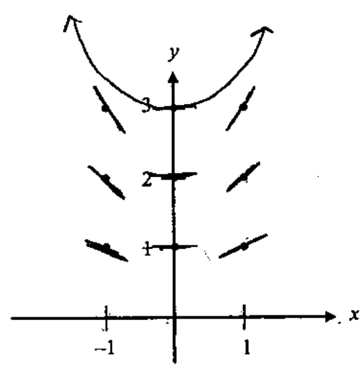
D $\tan x + 5$

E $\tan x + 5e^x$

29. Consider the differential equation given by $\dfrac{dy}{dx} = \dfrac{xy}{2}$.

a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

b) Draw a particular solution if $f(0) = 3$



c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

$\boxed{\approx 3.030}$

Ch. 6 REVIEW (NO PARTS)

① $\int \frac{3x}{\sqrt[3]{x^2+3}}\,dx$

Let $u = x^2+3$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{3}{2}\int \frac{du}{\sqrt[3]{u}}$

$= \frac{3}{2}\int u^{-1/3}\,du$

$= \frac{3}{2}\left(\frac{3}{2}u^{2/3}\right) + C$

$= \frac{9}{4}u^{2/3} + C$

$\boxed{= 9/4\,(x^2+3)^{2/3} + C}$

② $\int \sec x\,dx = \boxed{\ln|\sec x + \tan x| + C}$ MEMORIZE! (proof in class)

③ $\int \frac{x-1}{x+1}\,dx = \int\left(1 + \frac{-2}{x+1}\right)dx = \int 1\,dx + \int \frac{-2}{x+1}\,dx$

$$\begin{array}{r} 1 \\ x+1\,\overline{)\,x-1} \\ -(x+1) \\ \hline -2 \end{array}$$

$\boxed{= x - 2\ln|x+1| + C}$

if necessary, you can let $u = x+1$, $du = dx$

& $\int \frac{-2}{x+1}\,dx = -2\int \frac{du}{u} = -2\ln|u| + C$

④ $\int \sin x\,dx = \boxed{-\cos x + C}$ MEMORIZE!

⑤ $\int b^{4x}\,dx$

Let $u = 4x$

$du = 4dx \Rightarrow \frac{1}{4}du = dx$

$= \frac{1}{4}\int b^u\,du$

$= \frac{1}{4}\cdot\frac{1}{\ln b}\cdot b^u + C$

$\boxed{= \frac{1}{4\ln b}\cdot b^{4x} + C}$

⑥ $\int \tan x\,dx = \boxed{-\ln|\cos x| + C}$ MEMORIZE! (proof in class!)

⑦ $\int \cot^2 x\,dx = \boxed{-\cot x - x + C}$ MEMORIZE! (proof in class!)

(8) $\int \frac{(x+1)^2}{x^{1/3}}\,dx = \int \frac{x^2+2x+1}{x^{1/3}}\,dx = \int\left(\frac{x^2}{x^{1/3}} + \frac{2x}{x^{1/3}} + \frac{1}{x^{1/3}}\right)dx$

$= \int\left(x^{5/3} + 2x^{2/3} + x^{-1/3}\right)dx$

$= \frac{3}{8}x^{8/3} + 2\cdot\frac{3}{5}x^{5/3} + \frac{3}{2}x^{2/3} + C$

$\boxed{= \frac{3}{8}x^{8/3} + \frac{6}{5}x^{5/3} + \frac{3}{2}x^{2/3} + C}$

(9) $\int \cos^2 x\,dx = \boxed{\frac{\sin(2x)}{4} + \frac{x}{2} + C}$ MEMORIZE!

proof in class!

(10) $\int \frac{dx}{4+9x^2}$

Looks like a $\tan^{-1}x$ derivative... need to make the 4 a 1...

$\int \frac{dx}{4\left(1+\frac{9x^2}{4}\right)} = \frac{1}{4}\int \frac{dx}{1+\left(\frac{3x}{2}\right)^2}$

Let $u = \frac{3x}{2}$

$du = \frac{3}{2}\,dx \Rightarrow \frac{2}{3}\,du = dx$

$= \frac{1}{4}\cdot\frac{2}{3}\int \frac{du}{1+u^2}$

$= \frac{1}{6}\tan^{-1}(u) + C$

$\boxed{= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + C}$

(11) $\int \cos t\,dt = \boxed{\sin t + C}$ MEMORIZE!

(12) $\cancel{\int \sin^{-1}x\,dx}$ SKIP... Requires Integration by Parts

(13) $\int \sin^2 x\,dx = \boxed{\frac{x}{2} - \frac{\sin(2x)}{4} + C}$ MEMORIZE!

(14) $\int \frac{dx}{x\sqrt{x^2-4}}$

Looks like an inverse secant derivative ... need the 4 to be a 1

$$\int \frac{dx}{x\sqrt{4\left(\frac{x^2}{4}-1\right)}} = \frac{1}{2}\int \frac{dx}{x\sqrt{\left(\frac{x}{2}\right)^2-1}}$$

Let $u = \frac{x}{2} \Rightarrow 2u = x$

$du = \frac{1}{2}dx \Rightarrow 2du = dx$

$$= \frac{1}{2}\int \frac{\cancel{2}du}{\cancel{2}u\sqrt{u^2-1}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-1}}$$

$$= \frac{1}{2}\sec^{-1}|u| + C$$

$$\boxed{= \frac{1}{2}\sec^{-1}\left|\frac{x}{2}\right| + C}$$

(15) $\int \cot x\, dx = \boxed{\ln|\sin x| + C}$ MEMORIZE! (proof in class)

(16) $\int \csc x\, dx = \boxed{-\ln|\csc x + \cot x| + C}$ MEMORIZE! (proof in class)

(17) $\int \sqrt{x}\,(3-4x)\,dx = \int x^{1/2}(3-4x)\,dx = \int \left(3x^{1/2} - 4x^{3/2}\right)dx$

$$= 3\cdot\frac{2}{3}x^{3/2} - 4\cdot\frac{2}{5}x^{5/2} + C$$

$$\boxed{= 2x^{3/2} - \tfrac{8}{5}x^{5/2} + C}$$

(18) $\int \frac{dx}{\sqrt{-x^2-2x}} = \int \frac{dx}{\sqrt{-1(x^2+2x)}} = \int \frac{dx}{\sqrt{-1(x^2+2x\underbrace{+1)+1}_{\text{C.T.S.}}}} = \int \frac{dx}{\sqrt{1-(x+1)^2}}$

$\Rightarrow$ Let $u = x+1$, $du = dx$ $\Big\} = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C = \boxed{\sin^{-1}(x+1)^2 + C}$

(19) $\int \tan^2 x\,dx = \boxed{\tan x - x + C}$ MEMORIZE! (prof in class)

(20) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

Let $u = e^x$

$du = e^x dx$

$= \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$

$\boxed{= \sin^{-1}(e^x) + C}$

(21) $\int \csc^2 x\,dx = \boxed{-\cot(x) + C}$ MEMORIZE!

(22) $\int x e^{-x^2+3}\,dx$

Let $u = -x^2+3$

$du = -2x\,dx \Rightarrow -\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\int e^u\,du$

$= -\frac{1}{2}e^u + C$

$\boxed{= -\frac{1}{2}e^{-x^2+3} + C}$

(23) $\int \sec^2 x\,dx = \boxed{\tan x + C}$

(24) $\int \frac{3+x}{x^2+1}\,dx = \int \frac{3}{x^2+1}\,dx + \int \frac{x}{x^2+1}\,dx \Rightarrow$ Let $u = x^2+1$, $du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= 3\int \frac{1}{x^2+1}\,dx + \frac{1}{2}\int \frac{1}{u}\,du$

$= 3\tan^{-1}(x) + \frac{1}{2}\ln|u| + C$

$\boxed{= 3\tan^{-1}(x) + \frac{1}{2}\ln(x^2+1) + C}$

(25) $\int (\cos t - \sin t)^2\,dt = \int (\cos^2 t - 2\cos t\sin t + \sin^2 t)\,dt = \int 1 - 2\cos t\sin t\,dt$

$= \int (1 - \sin(2t))\,dt = \boxed{t - \frac{\sin(2t)}{2} + C}$ OR $\boxed{t - \sin^2 t + C}$

(26) $dy/dt = k \cdot y$

separate the variables

$$\frac{1}{y}\,dy = k \cdot dt$$

integrate

$$\int \frac{1}{y}\,dy = \int k \cdot dt$$

$$\ln|y| = kt + c$$

Solve for y

$$e^{kt+c} = y$$

$$e^{kt}\underbrace{e^{c}}_{\text{A constant!}} = y$$

$$Ce^{kt} = y$$

use initial conditions...

$$y(0) = y_0$$

$$Ce^{k(0)} = y_0$$

$$Ce^{0} = y_0$$

$$C = y_0$$

$$\boxed{\therefore\ y = y_0 e^{kt}}$$

(27) $dy/dx = \frac{3x^2+1}{2y}$

a) when $x=1$, $y=4$ [given $f(1)=4$] ... $dy/dx\Big|_{(1,4)} = \frac{3(1)^2+1}{2(4)} = \boxed{\frac{1}{2}}$

b) Equation of Line: pt $(1,4)$ Slope $= 1/2$

$$y-4 = \tfrac{1}{2}(x-1) \Rightarrow y = \tfrac{1}{2}(x-1)+4$$

$$f(1.2) \approx \tfrac{1}{2}(1.2-1)+4 = \tfrac{1}{2}(.2)+4 = \boxed{4.1}$$

27 continued...

ⓒ $dy/dx = \dfrac{3x^2+1}{2y}$

$2y\,dy = (3x^2+1)\,dx$

$\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

use initial condition $f(1)=4$

$4^2 = 1^3 + 1 + C$

$16 = 2 + C$

$14 = C$

$y^2 = x^3 + x + 14 \Rightarrow f(x) = \pm\sqrt{x^3+x+14}$

ⓓ $f(1.2) = \pm\sqrt{(1.2)^3 + (1.2) + 14} \approx \pm\sqrt{16.928}$

$\approx \boxed{\pm 4.114}$

NOTICE that's pretty close to the answer from part b.

(28) $dy/dx = y\sec^2 x$

$\frac{1}{y}\,dy = \sec^2 x\,dx$

$\int \frac{1}{y}\,dy = \int \sec^2 x\,dx$

$\ln|y| = \tan x + C$

$e^{\tan x + C} = y$

$Ce^{\tan x} = y$

when $x=0,\ y=5$

$Ce^{\tan(0)} = 5$

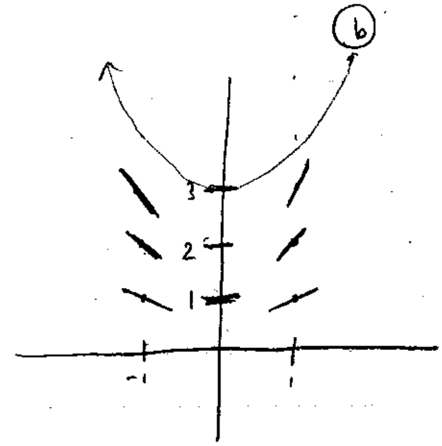
$C = 5$

$y = 5e^{\tan x}$

$\boxed{C}$ Answer

(29)

Pt	$dy/dx = \frac{xy}{2}$
(1,1)	1/2
(1,2)	1
(1,3)	3/2
(0,1)	0
(0,2)	0
(0,3)	0
(-1,1)	-1/2
(-1,2)	-1
(-1,3)	-3/2



(b) if $f(0) = 3$

(c) $\frac{dy}{dx} = \frac{xy}{2}$

$\int \frac{1}{y}\,dy = \int \frac{1}{2}x\,dx$

$\ln|y| = \frac{1}{4}x^2 + C$

$e^{\frac{1}{4}x^2 + C} = y$

$Ce^{x^2/4} = y$

if $f(0) = 3$, then $Ce^0 = 3$

$C = 3$

$y = 3e^{x^2/4} \Rightarrow f(x) = 3e^{x^2/4}$

$\therefore f(0.2) = 3e^{(.2)^2/4} \approx \boxed{3.030150501}$