1.2 FUNCTIONS AND GRAPHS

Functions

In the last section we discussed lines and when we needed to write "*y* as a function of *x*". But what is a function?
In Algebra 1, we defined a function as a rule that assigned one and only one (a unique) output for every input. We called the input the *domain* and the output the *range*.

Definition: Function

A **function** from a set *D* to a set *R* is a rule that assigns a unique element in *R* to each element in *D*.

Intervals

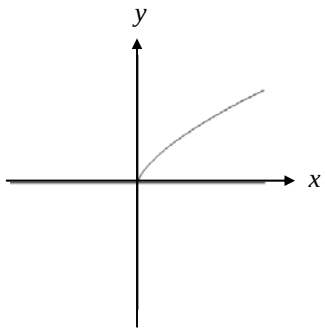
In this course we would like not only to know what the domain and range are, but how to describe them with the correct notation. The domain and range of a function could be all real numbers, or we may need to limit the domain and/or the range using *intervals* that are either *open* or *closed*.

Open and closed intervals have endpoints. If the **endpoint is included**, then we say the interval is **closed** at that point, and if the **endpoint is not included,** then we say the interval is **open** at that point. We use a parenthesis to indicate open and a bracket to indicate closed.

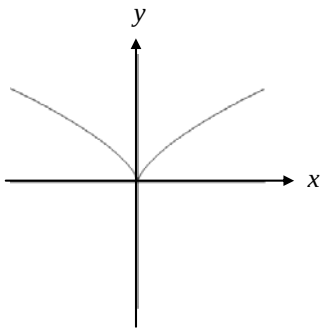
Example: Use interval notation AND inequality notation to describe each interval on the *x* – axes below.

Example: What are some things to look for in a function that might restrict the domain.

Example: Using the computer program Derive5, I graphed the equations below. What do you notice?



$y = x^{2/3}$



$y = \left(x^2\right)^{1/3}$

The purpose of the last example is for you to understand that while the calculator is a wonderful tool, it is not always perfect. For the purpose of this last example, your TI-83+ calculator will graph the function correctly either way. I personally find this interesting, since both the TI-83+ and the program Derive5 are produced by Texas Instruments.

**You should ALWAYS be aware of the domain of a function … especially when dealing with applications.

♬: While it is many times possible to just look for the restrictions to the domain, the range of a function is easier to determine if you have a graph or you know what the graph looks like. You should know what the following basic functions look like without having to use your calculator. Can you draw a quick sketch?

$y = x$	$y = x^2$	$y = x^3$	$y = \sqrt{x}$
$y = \frac{1}{x}$		$y = \frac{1}{x^2}$	
$y = \lvert x \rvert$	$y = \lfloor x \rfloor$	$y = ab^x,\ 0 < b < 1$	$y = ab^x,\ 1 < b$
$y = \log x$	$y = \sin x$	$y = \cos x$	$y = \tan x$

Even and Odd Functions

Recognizing the behavior of functions is not limited to their domain and range. Many functions have the symmetric property of being odd or even. You need to be able to recognize the graph of a function as odd or even, AND you need to understand how to show/verify/prove that a function is even or odd algebraically.

Graphical Recognition of Even and Odd Functions

An **EVEN** function is <u>symmetrical about the *y* – axis</u>. *Example*: $y = \cos x$

An **ODD** function is <u>symmetrical about the origin.</u> *Example*: $y = \sin x$

Algebraic Properties of Even and Odd Functions

An **EVEN** function has the property that $f(-x) = f(x)$.
That is, if you plug in "–*x*" into the function and simplify, you will obtain the original function.

An **ODD** function has the property that $f(-x) = -f(x)$.
That is, if you plug in "–*x*" into the function and simplify, you will obtain the opposite of the original function.

Example: Determine whether the following functions are even, odd, or neither.

a) $g(x) = x^3 - x$

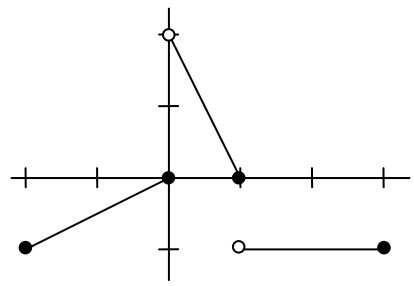
b) $h(x) = 1 + \cos x$

Piecewise Functions

Some functions are broken into pieces and behave differently depending on the restricted domain of each piece. Such functions are called piecewise functions. An example of a function that can be written as a piecewise function is the absolute value function $f(x)=|x|$.

Example: Sketch a picture of $f(x)=|x|$ and write an equation for the two "pieces" using a domain appropriate to each piece.

Example: Write a piecewise function for the graph at the right.



Composite Functions

When the range of one function is used as the domain of a second function we call the entire function a composite function. We use the notation $(f \circ g)(x)=f(g(x))$ to describe composite functions.
This is read as "*f* composed with *g*" or "*f* of *g* of *x*".

Example: If $f(x)=1-x^2$ and $g(x)=\sqrt{x}$, find $g(f(x))$. What is the domain and range of $g(f(x))$?

Example: If $f(x)=\dfrac{2x-1}{x+3}$, and $g(x)=\dfrac{3x+1}{2-x}$, find $f(g(x))$. Based on your answer, how might f and *g* be related?

Notecards from Section 1.2: Odd/Even Functions