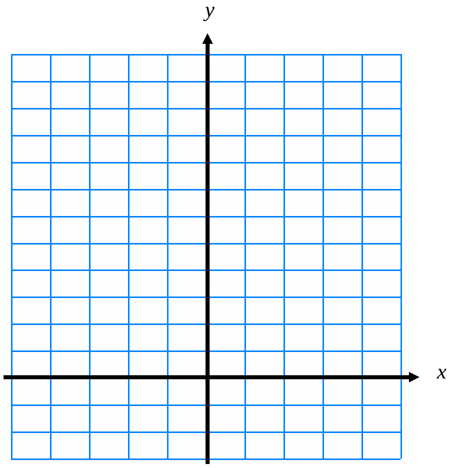
1.3 EXPONENTIAL FUNCTIONS

So far we've dealt with linear functions, piecewise functions, and composite functions. Next up, exponential functions.

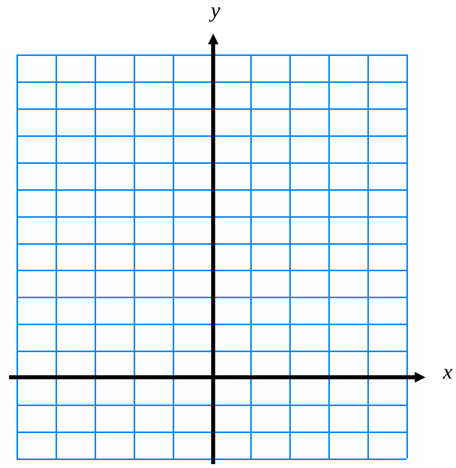
> *Definition*: *Exponential Function*
>
> If $b > 0$ and $b \neq 1$, then $f(x) = b^x$ is an exponential function with base *b*.

Example: Sketch the following graphs as accurately as possible on the graphs below:

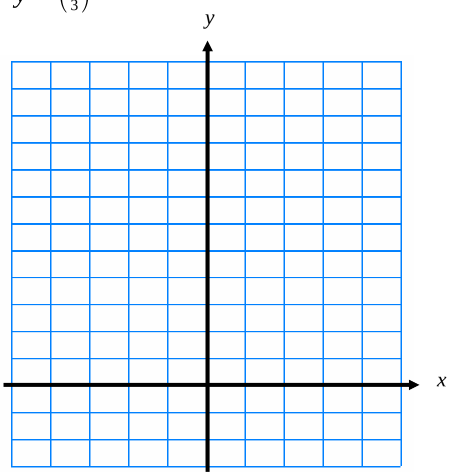
a) $y = 3^x$



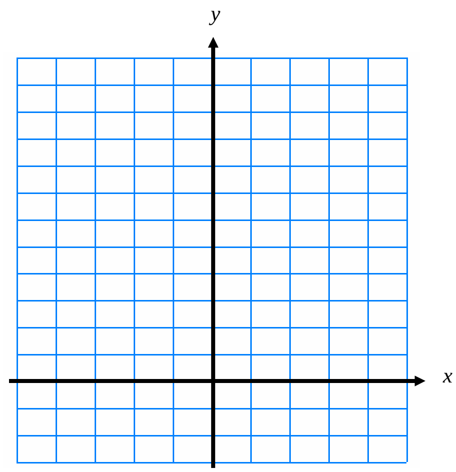
b) $y = 2 \cdot 3^x$



c) $y = \left(\frac{1}{3}\right)^x$



d) $y = 2 \cdot \left(\frac{1}{3}\right)^x$



Example: Which of these graphs show growth? decay?

Example: How does the 2 affect the graph?

Example: What is the domain and range of all 4 graphs?

Exponential Growth/Decay Model

In the exponential model $y = a \cdot b^x$, *b* is the rate of growth if *b* > 1, and *b* is the rate of decay if 0 < *b* < 1.
In either case, the initial value is *a*.

Example: Using your graphing calculator, let $Y_1 = x^2$ and $Y_2 = 2^x$. Graph both equations in the same window.

a) Solve the equation $x^2 = 2^x$ using your graphing calculator. Where are the solutions to this equation and how many are there?

b) Clear the two graphs from the screen and use the equation $x^2 - 2^x = 0$. Solve for *x* be graphing the left side of this equation. Where are the solutions to this equation and how many are there?

c) What did you learn from the last two questions?

The Number e

Many exponential functions in the real world are modeled using the base of *e*. Just like $\pi \approx 3.14$, we say $e \approx 2.718$. We can also define *e* using the function $\left(1+\frac{1}{x}\right)^x$ as follows:

$$\text{As } x \to \infty, \ \left(1+\tfrac{1}{x}\right)^x \to e$$

Example: Suppose you invest $12,000 in an account that earns you 5% interest compounded monthly for 10 years.

a) What is the initial amount?

b) What is the growth rate?

c) How many times does your money grow in 10 years? (How many times is interest added to your account?)

d) How much money will you have in 10 years?

e) Suppose the interest was compounded continuously. How much would you have in 10 years?

Example: You buy a brand new car for $35,000 and find out it depreciates at 12.5% per year. Write an exponential equation modeling this situation. How much will your car be worth in 5 years?

Example: The half – life of Ra – 226 is 1,620 years. If there are 10g initially, how much Ra – 226 is left after 1,000 years?

Example: The number of United States citizens *y* (in millions) who traveled to foreign countries in the years 1988 through 1996 are shown in the table below., where *t* = 8 represents the year 1988.

t	8	9	10	11	12	13	14	15	16
y	40.7	41.1	44.6	41.6	43.9	44.4	46.5	50.8	52.3

a) Use the regression capabilities of your graphing calculator to find an exponential model that fits the data.

b) According to the model, is the number of travelers increasing or decreasing? At what rate?

c) Using your model, how many travelers were there in 1980? 1974? 2006?

d) Why is it important to let *t* = 8 represent the year 1988? [Try answering question *c* using the actual year.]