

1.4 PARAMETRIC EQUATIONS

Up to this point all the functions we have been looking at have used a single equation with two variables, x and y . In this section we use a third variable to represent the curve. This third variable is called a parameter.

Example: Using your graphing calculator, set your window to $X_{\min} = -5$, $X_{\max} = 80$, $X_{\text{sc1}} = 5$,
 $Y_{\min} = -5$, $Y_{\max} = 20$, $Y_{\text{sc1}} = 5$.

Graph $y = -\frac{x^2}{72} + x$, which models the path of an object thrown into the air at a 45° angle at an initial velocity of 48 feet per second. (Just take my word for it ☺)

Use **TRACE** to locate a few points on the graph. What do you learn about the object from this information?

Example: Now change your calculator to *parametric* mode and plot $x_1 = 24\sqrt{2}t$.
 $y_1 = -16t^2 + 24\sqrt{2}t$

Notice it is the same graph ... more on that later.

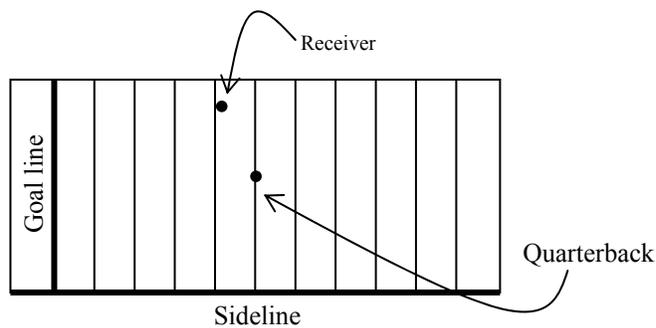
Use **TRACE** to locate a few points on the graph. What do you learn about the object from this information that you did not know before?

Example: During a football game, the quarterback held the ball on the 50 yard line while the receiver ran toward the goal line. After waiting 3 seconds, the quarterback threw the ball to the receiver.

Using the intersection of the goal line and the sideline as the origin, let x = the number of yards from the goal line, y = the number of yards from the sideline, and t = the number of seconds the receiver has been running.

The two equations describing the receiver's path are given by $x_1 = 42 - 6t$ $0 \leq t \leq 7.5$
 $y_1 = 45 - t$

The two equations describing the path of the ball are given by $x_2 = 50 - 22(t-3)$ $3 \leq t \leq 7.5$
 $y_2 = 27 + 6(t-3)$



**These equations give the path of the ball viewed from above. They ignore the height of the ball.

a) Choose an appropriate window and graph the receiver's path. Explain your choice for the window.

b) Graph the path of the ball.

♣: To make sure that the graph doesn't start for 3 seconds, enter $x_2 = 50 - 22(t-3)(t \geq 3)$
 $y_2 = 27 + 6(t-3)(t \geq 3)$

c) Assuming the height of the ball is not an issue, does the receiver catch the ball? Explain your reasoning. If not, change the equations above so that the receiver does catch the ball.

Definition: Parametric Curve

If x and y are given as functions

$$x = f(t), \quad y = g(t)$$

over an interval of t values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve.

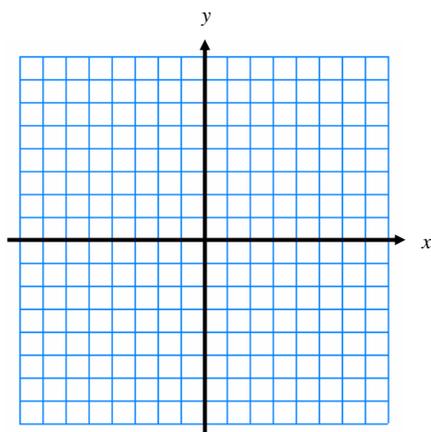
Graphing Parametric Curves Without a Calculator

Just like when you learned to graph for the first time back in Algebra 1, we are going to make a table of values. The difference is that we now have three variables instead of two.

Example: Graph the parametric curve

$$x = t^2 - 4 \quad -2 \leq t \leq 3$$

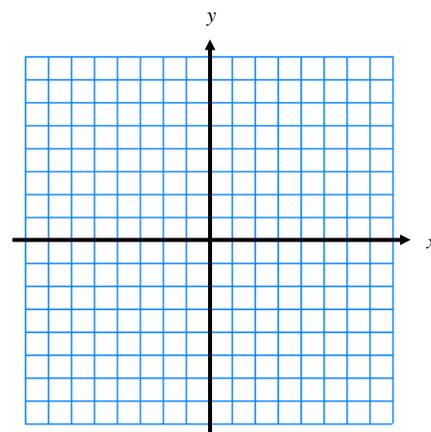
$$y = \frac{t}{2}$$



Example: Graph the parametric curve

$$x = 4t^2 - 4 \quad -1 \leq t \leq \frac{3}{2}$$

$$y = t$$



Example: Compare and contrast the two graphs above.

Changing from Parametric to Rectangular (Cartesian)

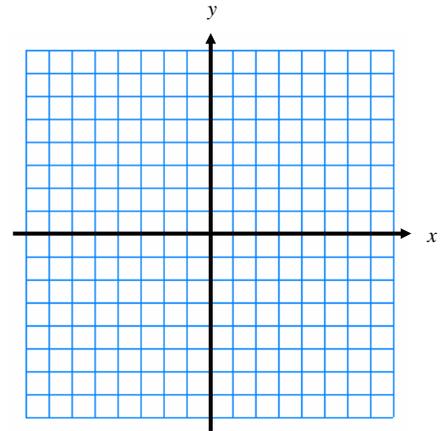
To change a parametric equation back into a more familiar rectangular (Cartesian) equation you must eliminate the parameter. The typical approach to doing this is to solve for the parameter in one of the equations and then simply substitute that solution into the other equation.

Example: Change the parametric equation defined by $x = \sqrt{t}$ into a Cartesian equation.

$$y = 2 - t$$

The next two examples illustrate another way to eliminate the parameter by using trigonometric identities.

Example: Graph $x = 3 \cos \theta + 2$ for $0 \leq \theta \leq 2\pi$, then change the parametric equation into a Cartesian equation.
 $y = 5 \sin \theta - 1$



Example: Change the parametric equation defined by $x = 4 \sec \theta$ into a Cartesian equation. Can you identify the graph of this equation without graphing it?
 $y = 3 \tan \theta$

Writing a Parametric Equation

Changing from Rectangular to Parametric means you get to create a parameter. As the graphs on the previous page indicated, your choice of parameter should not change the shape of the graph, only the “speed” in which the graph is drawn.

Example: Find a parametrization for the left half of the parabola $y = x^2 + 2x$.

Example: Find a parametrization for the line segment with endpoints $(-1, 3)$ and $(3, -2)$.