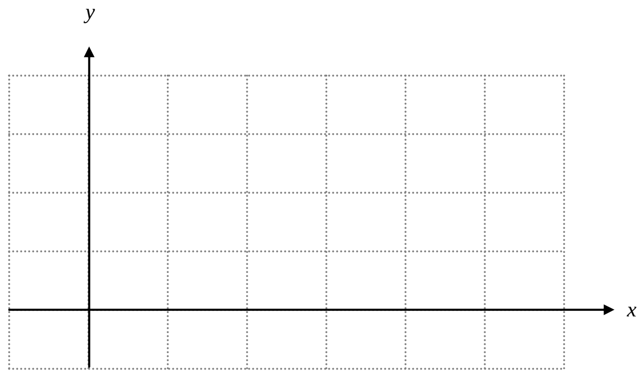
3.2 DIFFERENTIABILITY

The focus on this section is to determine when a function fails to have a derivative.

Example: Using the grid provided, graph the function $f(x) = |x-3|$.



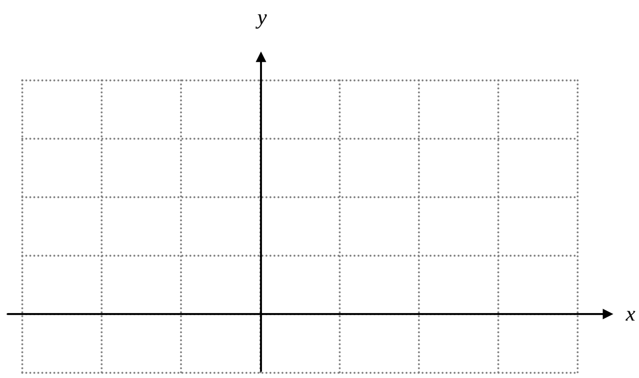
a) What is $f'(x)$ as $x \to 3^-$?

b) What is $f'(x)$ as $x \to 3^+$?

c) Is *f* continuous at *x* = 3?

d) Is *f* differentiable at *x* = 3?

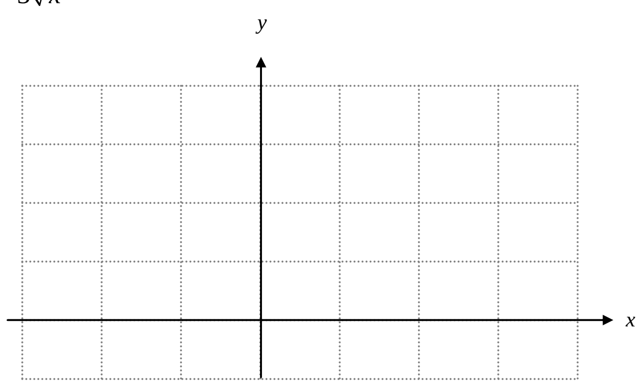
Example: Graph $f(x) = x^{2/3}$



a) Describe the derivative of *f* (*x*) as *x* approaches 0 from the left and the right.

b) Suppose you found $f'(x) = \dfrac{2}{3\sqrt[3]{x}}$. What is the value of the derivative when *x* = 0?

Example: Graph $f(x) = \sqrt[3]{x}$



a) Describe the derivative of *f* (*x*) as *x* approaches 0 from the left and the right.

b) Suppose you found $f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$. What is the value of the derivative when *x* = 0?

The three examples above along with any graph that is not continuous are not differentiable. The first graph had a "corner" or a sharp turn and the derivatives from the left and right did not match. The second graph had a "cusp" where secant line slope approach positive infinity from one side and negative infinity from the other. The third graph had a "vertical tangent line" where the secant line slopes approach positive or negative infinity from both sides.

In all three of the previous examples the functions were continuous, but failed to be differentiable at certain points. Continuity does not guarantee differentiability, but it does work the other way around.

Theorem 2.1 Differentiability Implies Continuity

If f is differentiable at $x = c$, then f is continuous at $x = c$.

Example: The converse of this statement is NOT true! What is the Converse?

Example: The contrapositive of any statement is logically equivalent to the original statement. What is the Contrapositive to this statement?

Using the TI – 83+

Most graphing calculators can take derivatives at certain points. However, they use a different method of calculating the derivative than our earlier definitions.

Example: Provide a geometric interpretation of this formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example: Draw a picture to represent this formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

Your graphics calculator uses the concept of this last definition to calculate derivatives.

To use your graphics calculator to find the derivative, use the `nDeriv(` function on the TI-83+. To access this function press MATH, then 8 (or use ↑ and ↓ to go to `nDeriv(` and press ENTER). The `nDeriv(` function works as follows:

```
nDeriv(function,variable,value)
```

Where "function" is the function you want to find the derivative of, "variable" is the variable you are differentiating with respect to, and "value" is the point at which you want to find the derivative.

Example: Use your calculator to find the derivative of $f(x) = x^2 - 3x + 2$ at $x = -3$.

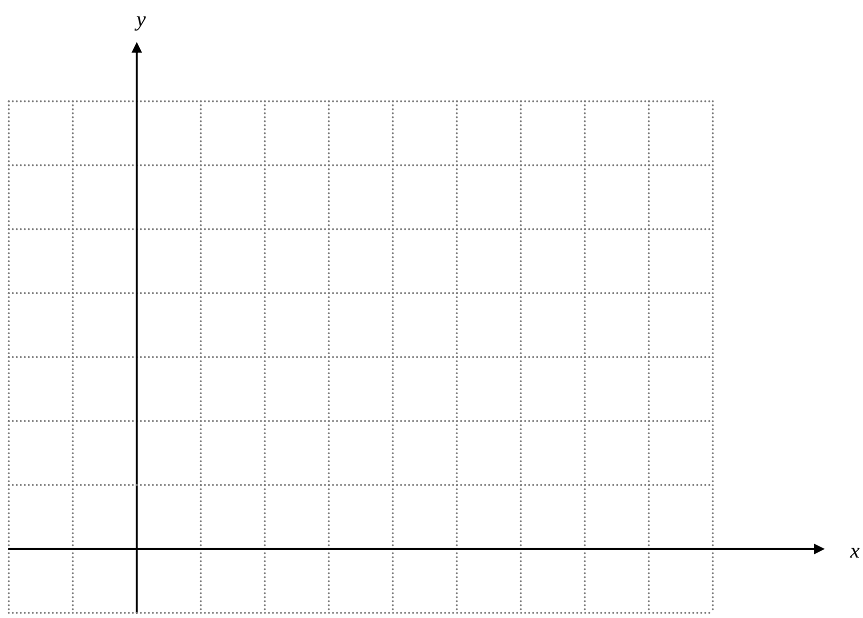
Example: Use your calculator to find the derivative of the three examples at the beginning. What problems do you find?

Example: Find the derivative of $f(x) = \left|x^2 - 3x + 2\right|$ at $x = 2$.

a) Using `nDeriv(`

b) Using the GRAPH screen.
Press Y= , and enter the equation.
Press GRAPH.
Press 2ND , TRACE … (`Calc`)
Press 6: … (`dy/dx`)
Press 2 … (this defines $x = 2$ and the calculator calculates the derivative at $x = 2$)

c) Graph the function by hand on the grid below. Based on your graph, what is $f'(2)$?



Example: Find the derivative of $f(x) = \dfrac{1}{x}$ at $x = 0$.

(a) Using `nDeriv(`

(b) Using the definition of a derivative.

Example: Find the equation of the tangent line to the graph of $f(x) = x^3 + x^2$ when $x = 2$.