3.3 RULES FOR DIFFERENTIATION

Drum Roll please … [In a Deep Announcer Voice] … And now … the moment YOU'VE ALL been waiting for …

Rule #1 *Derivative of a Constant Function*

If c is any constant value, then $\frac{d}{dx}[c]=0$

This should not be too earth shattering to you, since the slope of a constant function is always 0!

Example: Let $f(x)=5$. Find $f'(x)$.

Rule #2 *Power Rule*

If n is any number, then $\frac{d}{dx}\left[x^n\right]=n\cdot x^{n-1}$, provided x^{n-1} exists.

♬ : In section 3.3 your book distinguishes between *n* being a positive integer (rule 2), *n* being a negative integer (rule 7) and *n* being a rational number (rule 9, section 3.7). The distinction is made so that they may prove each separate case in the book. However, the use of the power rule is unchanged for all three different values of *n*.

Example: Prove the Power Rule for positive integer values of *n*.

Example: Let $f(x)=x^5$. Find $f'(x)$.

Example: Let $f(x)=\sqrt[3]{x^2}$. Find $f'(x)$.

Example: Let $f(x)=\frac{1}{x^4}$. Find $f'(x)$.

Rule 3: *The Constant Multiple Rule*

If u is a differentiable function of x and c is a constant, then

$$\frac{d}{dx}[cu] = c\frac{du}{dx}.$$

Example: Let $y = 5x^7$. Find $\frac{dy}{dx}$.

Example: Let $g(x) = \frac{4}{5x^3}$. Find $g'(x)$.

Rule 4: *The Sum and Difference Rule*

If u and v are differentiable functions of x, then wherever u and v are differentiable

$$\frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$$

Example: Let $y = x^3 + 4x^2 - 2x + 7$. Find y'.

Example: Let $g(x) = \frac{3}{(-2x)^4} - \frac{x}{2} + \frac{1}{4}$. Find $g'(x)$.

Example: Find the equation of the tangent line to the function $f(x) = 4x^3 - 6x + 5$ when $x = 2$.

Example: Find all points where the graph of $y = x^4 - 5x^3 - 3x^2 + 13x + 10$ has a horizontal tangent line.

Example: Let $h(x) = (x^2 + 1)(2x - 5)$. Find $h'(x)$.

Example: The volume of a cube with sides of length s is given by $V = s^3$. Find $\frac{dV}{ds}$ when $s = 4$ centimeters.

Using Rule 4, we know that the derivative of the sum of two functions is the sum of the derivatives of the two functions. This does not work for the product and quotient of two functions. To illustrate this, we look at the following example.

Example: Find $\frac{d}{dx}[x \cdot x]$.

<u>Rule 5</u>: *The Product Rule*

If u and v are differentiable functions of x, then

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$

This is also written as

$$\frac{d}{dx}[uv] = uv' + vu'$$

For polynomial functions it is not always necessary to use the product rule, however, with trigonometric, exponential, logarithmic, and other functions, it is a necessary tool.

Example: Let $f(x) = (3x^3 + 4x^2)(2x^4 - 5x)$. Find $f'(x)$ without using the product rule first, then using the product rule.

Example: Let $y = (3 + 2\sqrt{x})(5x^3 - 7)$. Find $\frac{dy}{dx}$.

Example: Find the equation of the tangent line to the graph of $f(x) = (x^3 - 3x + 1)(x + 2)$ at the point (1, –3).

Rule 6: *The Quotient Rule*

If u and v are differentiable functions of x, and $v \neq 0$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

This is also written as

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}.$$

Looks GREAT! Doesn't it?! Well, luckily for you, it ain't that bad. With thanks to Snow White and the Seven Dwarfs, if we replace u with *hi* and v with *ho* (*hi* for high up there in the numerator and *ho* for low down there in the denominator), and letting D stand for "the derivative of", the formula becomes

$$D\left(\frac{hi}{ho}\right) = \frac{ho\,D(hi) - hi\,D(ho)}{(ho)^2}$$

In words, that is "**ho dee hi minus hi dee ho over ho ho**". Now, if Sleepy and Sneezy can remember that, it shouldn't be any problem for you.

Example: Find $\frac{d}{dx}\left(\frac{x}{x^2+1}\right)$

Example: Find $\frac{d}{dx}\left[\frac{5x^2}{x^3+1}\right]$

Example: For $a - d$, find $f'(2)$ given the following information:

$$g(2) = 3 \quad g'(2) = -2$$
$$h(2) = -1 \quad h'(2) = 4$$

a) $f(x) = 2g(x) + h(x)$

b) $f(x) = 4 - h(x)$

c) $f(x) = g(x)h(x)$

d) $f(x) = \frac{g(x)}{h(x)}$

Second and Higher Order Derivatives

The first derivative of y with respect to x is denoted y' or $\frac{dy}{dx}$. The second derivative of y with respect to x is denoted y'' or $\frac{d^2y}{dx^2}$. The second derivative is an example of a higher – order derivative. We can continue to take derivatives (as long as they exist) using the following notation:

First derivative	y'	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$
Second derivative	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$
Third derivative	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}[f(x)]$
Fourth derivative	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}[f(x)]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n^{th} *derivative*	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}[f(x)]$

Example: Find $\frac{d^4}{dx^4}\left[-5x^8+2x^6-9x^3+32x-1\right]$.

Example: Let $f(x)=\frac{x}{x-1}$. Find $f''(x)$.

Example: If $f^{(4)}(x)=2\sqrt{x}$, find $f^{(5)}(x)$.