3.4 VELOCITY AND OTHER RATES OF CHANGE

Instantaneous Rates of Change

We have already seen that the instantaneous rate of change is the same as the slope of the tangent line and thus the derivative at that point. Unless we use the phrase "average rate of change", we will assume that in calculus the phrase "rate of change" refers to the instantaneous rate of change.

Example: The length of a rectangle is given by $2t + 1$ and its height is $\sqrt{t}$, where t is time in seconds and the dimensions are in centimeters. Find the rate of change of the area with respect to time, and indicate the units of measure for this rate.

Example: Boyle's Law states that if the temperature of a gas remains constant, its pressure is inversely proportional to its volume. Show that the rate of change of the pressure is inversely proportional to the square of the volume.

Motion along a line

When a spring attached to a wall is stretched and then released, it moves back and forth. This motion can be described using functions involving sine and cosine (which we are not ready for just yet …). However, motion along a line in other circumstances (either horizontal or vertical lines) can also be described using functions. Typically, we use a position function $s(t)$ to describe the position s of an object after t seconds.

<u>Important Vocabulary</u>:

The *displacement* of an object is the TOTAL CHANGE IN POSITION.

The *average velocity* of the object is described as TOTAL CHANGE IN POSITION (displacement) divided by the TOTAL CHANGE IN TIME.

The i*nstantaneous velocity* of the object is the derivative of the position function. Unless we use the term "average velocity", we will assume that velocity refers to instantaneous velocity.

Speed is the absolute value of velocity. Thus speed is a positive value, whereas, velocity indicated direction.

Acceleration is the rate of change in velocity. Thus, acceleration is the derivative of velocity. Since it is the derivative of velocity, it is also the *second* derivative of position.

Example: Bugs Bunny has been captured by Yosemite Sam and forced to "walk the plank". Instead of waiting for Yosemite Sam to finish cutting the board from underneath him, Bugs finally decides just to jump. Bugs' position, s, is given by

$$s(t) = -16t^2 + 16t + 320$$

where s is measured in feet and t is measured in seconds.

a) When will Bugs hit the ground?

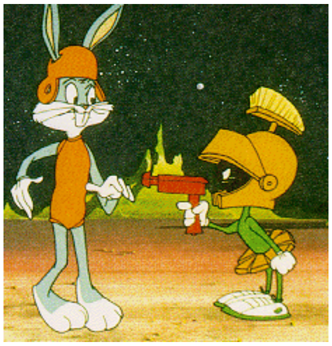
b) What is Bugs' velocity at impact? (What are the units of this value?)

c) What is Bug's speed at impact?

d) Find Bug's acceleration as a function of time. (What are the units of this value?)

Example: Once again trying to blow up earth because it interferes with his view of Venus, Marvin the Martian lands on the moon. Bugs Bunny, as always, interferes with his plan. Chasing Bugs, Marvin fires a warning shot straight up into the air with his Acme Disintegration Pistol. The height (in feet) after t seconds of the shot is given by

$$s(t) = -2.66t^2 + 135t + 3.$$

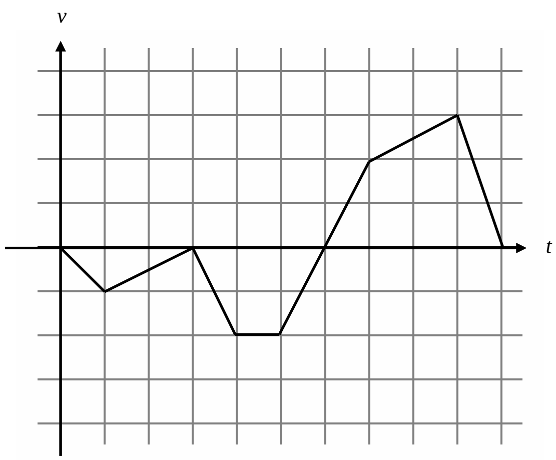


a) Find the velocity and acceleration as functions of time. (What is the meaning of the acceleration function?)

b) How long will it take for Marvin's shot to reach its maximum height?

c) What is the maximum height for Marvin's shot?

Example: Suppose the graph below shows the velocity of a particle moving along the x – axis.



a) Which way does the particle move first?

b) When does the particle stop?

c) When does the particle change direction?

d) When is the particle speeding up?

e) When is the particle slowing down?

f) When is the particle moving at a constant speed?

g) Graph the particle's speed for $0 \le t \le 10$.

h) Graph the particle's acceleration for $0 < t < 10$.