3.5 DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

The goal for this lesson is to introduce the derivatives of the 6 trigonometric functions. It is vital to your success in this course that you commit all 6 of these to memory as quickly as possible!

We are going to prove the derivative of sine. Since the proof involves the limit definition of the derivative, we will first need to discuss the limit of two very important functions, the first of which you should already know.

Example: $\lim_{x\to 0}\frac{\sin x}{x}$

Example: Investigate $\lim_{x\to 0}\frac{\cos x-1}{x}$.

Example: Find $\lim_{x\to 0}\frac{\cos x-1}{x}$ by multiplying the top and bottom by $\cos x+1$.

Example: Find the derivative of sin *x* using the limit definition of the derivative.

You can prove $\frac{d}{dx}[\cos x]=-\sin x$ using the same method and the same two limits above. Use the quotient rule for the next example.

Example: Find the derivative of tan *x* using the derivatives of sin *x* and cos *x*.

Using the derivatives of sin *x* and cos *x*, you can find the derivatives of the other 3 trig functions as well.

Derivatives of the Six Basic Trigonometric Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

We now have the power rule, the product rule, the quotient rule, and the derivatives of all 6 trig functions at our disposal.

Example: Find the derivative of each function. Before you begin, state which rule(s) you are going to have to use.

a) $f(x) = x^2 \sin x$

b) $f(x) = \frac{\cos x}{x}$

c) $g(t) = \sqrt{t} + 4\sec t$

d) $h(\theta) = 5\sec\theta + \tan\theta$

e) $h(s) = \frac{1}{s} - 10\csc s$

f) $y = x\cot x$

Example: If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

Example: If $f(x) = \sin x$, find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$. What do you think the function $f^{(100)}(x)$ is?

Using the TI – 83+

You can use your calculator to graph the derivative for you using the procedures outlined below.

We will be using the `nDeriv(` function, except we will be using it to define a function under Y_1. Remember the syntax is

`nDeriv(function,variable,value)`

For our example, let's use cos *x*. The difference will be that our function will be entered as a function of any variable other than *x*, and differentiated with respect to that same variable, and evaluated at the value of *x* instead of an actual number.

So, we want to enter the following:

$Y_1 =$ `nDeriv(cos(T),T,X)`

Step 1: Press [Y=].

Step 2: To enter `nDeriv(` … Press [MATH] then 8: `nDeriv(`

Step 3: Enter the function using the [ALPHA] key to enter a variable other than *x*. I chose *T*, but it should work with any letter you choose.

Step 4: After entering a comma, enter the same letter from step 3 as the variable you want to take the derivative with respect to. Again, I chose *T*, it's completely your choice.

Step 5: Enter the last comma, and then push the [X,T,Θ,n] button to enter the *X*. Do NOT use the [ALPHA] key to enter the *X* or it will not work.

Step 6: Graph the function by pressing [GRAPH].

Example: Graph the derivative of $f(x) = \ln x$. What function does this look like? Graph your guess on the same screen.

Example: Graph the derivative of $f(x) = e^x$. What function does this look like? Graph your guess on the same screen.