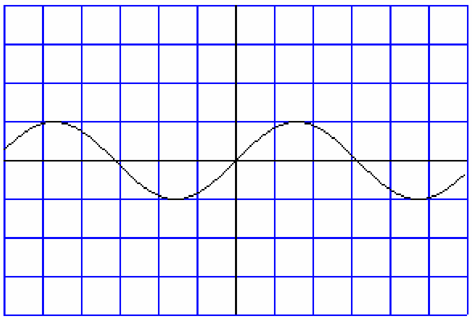
3.8 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Due to the periodic property of trigonometric functions, all 6 of the trig functions fail the horizontal line test (test for whether or not an inverse exists). In the graphs below, color (or highlight) the appropriate part of the trig function to use for its inverse, then graph it's inverse function.

Each box on each grid represents 1. You may wish to put appropriate values of π on the grid.

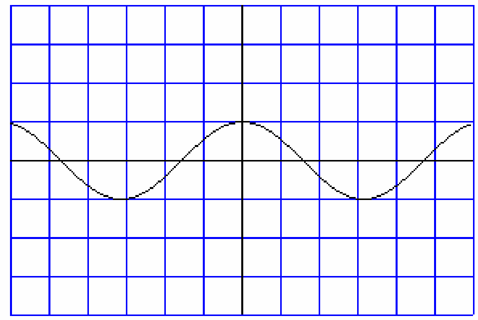
$y = \sin x$



Domain of Inverse:

Range of Inverse:

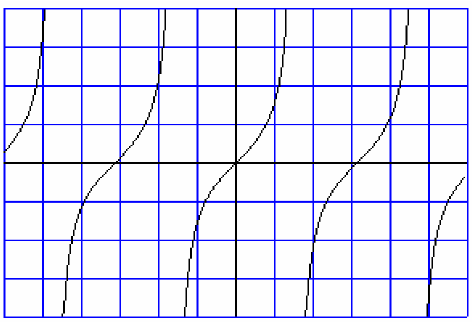
$y = \cos x$



Domain of Inverse:

Range of Inverse:

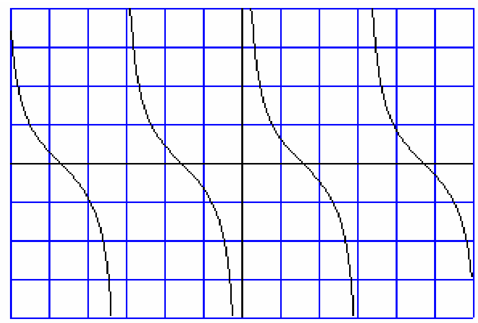
$y = \tan x$



Domain of Inverse:

Range of Inverse:

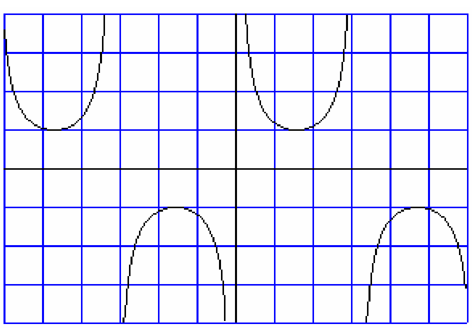
$y = \cot x$



Domain of Inverse:

Range of Inverse:

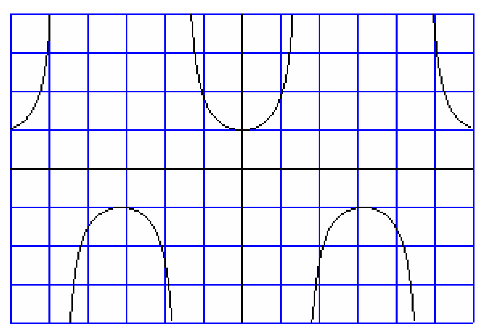
$y = \csc x$



Domain of Inverse:

Range of Inverse:

$y = \sec x$



Domain of Inverse:

Range of Inverse:

Properties of Inverse Trigonometric Functions

* Domains are restricted to make them functions

* $\sin^{-1}(x)$ and $\arcsin(x)$ are the same thing

Derivatives of Inverse Trigonometric Functions

1. $\frac{d}{dx}\left[\sin^{-1}(u)\right]=\frac{u'}{\sqrt{1-u^2}}$
2. $\frac{d}{dx}\left[\cos^{-1}(u)\right]=\frac{-u'}{\sqrt{1-u^2}}$
3. $\frac{d}{dx}\left[\tan^{-1}(u)\right]=\frac{u'}{1+u^2}$
4. $\frac{d}{dx}\left[\cot^{-1}(u)\right]=\frac{-u'}{1+u^2}$
5. $\frac{d}{dx}\left[\sec^{-1}(u)\right]=\frac{u'}{|u|\sqrt{u^2-1}}$
6. $\frac{d}{dx}\left[\csc^{-1}(u)\right]=\frac{-u'}{|u|\sqrt{u^2-1}}$

Example: Prove $\frac{d}{dx}\left[\sin^{-1}(x)\right]=\frac{1}{\sqrt{1-x^2}}$

Example: Prove $\frac{d}{dx}\left[\tan^{-1}(x)\right]=\frac{1}{1+x^2}$

Example: Prove $\frac{d}{dx}\left[\sec^{-1}(x)\right]=\frac{1}{|x|\sqrt{x^2-1}}$

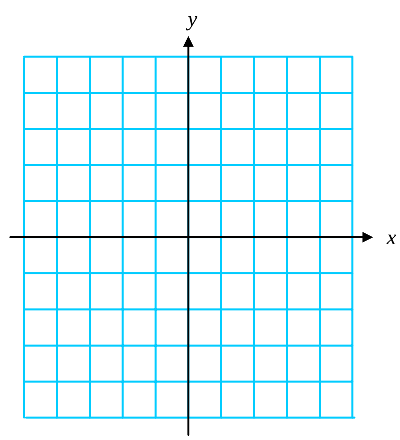
Example: Find the derivative of $f(t) = \sin^{-1}(t^2)$

Example: Find the derivative of $h(x) = x\sec^{-1}(x)$

Example: Find the derivative of $f(x) = x\sqrt{1-x^2} + \cos^{-1}(x)$

Example: Find the derivative of $y = \tan^{-1}\left(\sqrt{x-1}\right)$

Example: Graph the line $y = 4x + 1$



a) What is the slope of the line?

b) Graph the inverse function.

c) What is the slope of the inverse function?

d) If (2, 9) is on the original line, what point does it correspond to on the inverse function?

The slope of the line at (2, 9) on the original function is the _______________ of the slope of the inverse. The difference is that the slope of the inverse is calculated using the point ___________ instead of (2, 9).

New Notation

We have a different notation is used to describe the derivative of f at the point $x = a$. We can write

$$\left.\frac{df}{dx}\right|_a$$

Using the concepts from the last example, we can find the derivative of the inverse function without actually finding the inverse function. We will use the notation above to describe the relationship.

Derivative of the inverse function at a point (p, q) … this implies the point (q, p) is on the original function.

To find the derivative of f^{-1} at the point (p, q) we find the reciprocal of the derivative of f at the point (q, p).

$$\left.\frac{df^{-1}}{dx}\right|_p = \frac{1}{\left.\frac{df}{dx}\right|_q}$$

♬: Notice that the point at which you evaluate the derivative of the inverse is NOT the same point at which you evaluate the derivative of the function.

Example: Let $f(x) = x^5 + 2x - 1$. Verify (0, –1) is on the graph. Find $\left(f^{-1}\right)'(-1)$.

Example: Let $f(x) = x^3 + 2x - 1$. Find $\left.\frac{df^{-1}}{dx}\right|_2$.