

3.9 DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONSDerivative of $f(x) = e^x$

$$\frac{d}{dx}[e^x] = e^x$$

Proof: Prove this derivative using the limit definition of the derivative and the fact that $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$.

The Chain Rule and $f(x) = e^x$ If u is a differentiable function of x , then

$$\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}.$$

Example: Find $\frac{d}{dx}[e^{2x-1}]$

Example: Find $\frac{d}{dx}[e^{-3/x}]$

Example: Suppose $10 = e^{xy} + x^2 + y^2$, find $\frac{dy}{dx}$.

Example: Find $g'(t)$ if $g(t) = t^e(e^{-t})$

Derivative of $f(x) = \ln x$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Proof:

The Chain Rule and $f(x) = \ln x$

If u is a differentiable function of x , then

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx} \quad \dots \text{ or } \dots \quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Example: Let $y = \ln(2x+2)$. Find y' .

Example: Let $f(x) = \ln(\tan x)$. Find $f'(x)$

Example: Find $g'(t)$ if $g(t) = \ln(\ln t)$.

We can use the properties of logarithms to simplify some problems. Here's a quick refresher on those properties.

Definition of a logarithm: $\log_b a = c \Leftrightarrow b^c = a$

3 Rules of Logarithms: 1. $\log_b(MN) = \log_b(M) + \log_b(N)$

2. $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

3. $\log_b(M^k) = k \cdot \log_b(M)$

Change of Base Formula: $\log_b a = \frac{\log a}{\log b}$ or $\frac{\ln a}{\ln b}$

Example: Use the properties of logarithms to rewrite the function, then find the derivative of $y = \log_5 \sqrt{x}$.

Example: Find $h'(x)$ if $h(x) = \ln\left(\frac{1+e^x}{1-e^x}\right)$.

Logarithmic Differentiation

Example: Find $\frac{dy}{dx}$ if $y = 2^x$.

Example: Find $\frac{dy}{dx}$ if $y = 3^x$.

Example: Make a conjecture on $\frac{d}{dx}[a^x]$, where a is a constant greater than 0 and not equal to 1.

Derivative of a^u

If $a > 0$ and $a \neq 1$ and u is a differentiable function of x , then

$$\frac{d}{dx}[a^u] = a^u \ln a \cdot \frac{du}{dx}$$

Example: Use the technique of logarithmic differentiation to find $\frac{dy}{dx}$ for $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$.

Example: Find the first derivative for $y = x^{\ln x}$

Example: Find y' if $y = \frac{x^3}{3^x}$.