6.1 ANTIDERIVATIVES AND SLOPE FIELDS

Indefinite Integrals

In the previous chapter we dealt with definite integrals. Definite integrals had limits of integration . Indefinite integrals do not.

The set of all antiderivatives of a function $f(x)$ in the **indefinite integral of f with respect to x** and is denoted $$\int f(x)\,dx$$

Recall, that all antiderivatives differ by a constant, so if $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$, where C is the constant of integration. The following table gives a list of results you should already be familiar with.

Integral Formulas

1. Power Rule for $n \neq -1$: $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$
2. Rule for $n = -1$: $\int \dfrac{1}{x}\,dx = \ln|x| + C$
3. $\int e^{kx}\,dx = \dfrac{e^{kx}}{k} + C$
4. $\int \sin(kx)\,dx = \dfrac{-\cos(kx)}{k} + C$
5. $\int \cos(kx)\,dx = \dfrac{\sin(kx)}{k} + C$
6. $\int \sec^2(x)\,dx = \tan(x) + C$
7. $\int \csc^2(x)\,dx = -\cot(x) + C$
8. $\int \sec(x)\tan(x)\,dx = \sec(x) + C$
9. $\int \csc(x)\cot(x)\,dx = -\csc(x) + C$

Example: Evaluate each integral.

a) $\int \left(-x^{-3} + \sqrt[3]{x} - 1 + e^{3x}\right) dx$

b) $\int (3\sin x - \sin 3x)\,dx$

Differential Equations

A **differential equation** is an equation containing a derivative. Just like in Algebra, when you want to solve an equation, you use an inverse operation. To "undo" a derivative we take an _______________________________________.

Recall, that a function can have many antiderivatives, all of which vary by a _________________________.

Solving a differential equation involves finding a unique equation that satisfies some *initial conditions* or *initial values*.

The **order** of a differential equation is the order of the highest derivative involved in the equation.

Example: Solve $\dfrac{dy}{dx} = \sin x$ by **separation of variables** if $y(0) = 2$.

A Graphical Look at Differential Equations

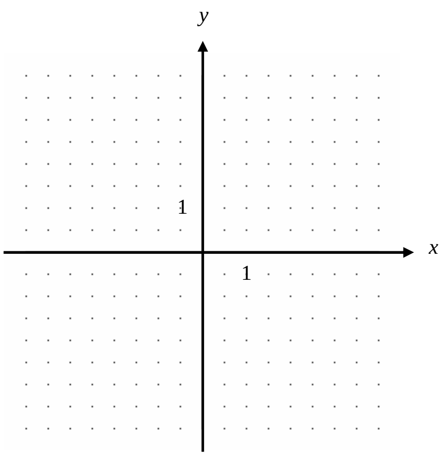
A **slope field** (or direction field) for the first order differential equation $\frac{dy}{dx} = f(x, y)$ is a plot of short line segments with slope $f(x, y)$ for a lattice of points (x, y) in the plane.

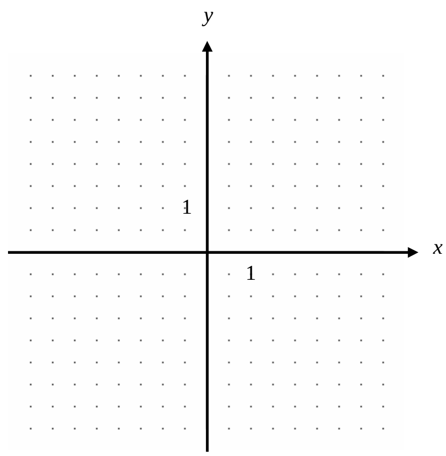
Example: On the diagram below, plot the slope field of the indicated differential equation.

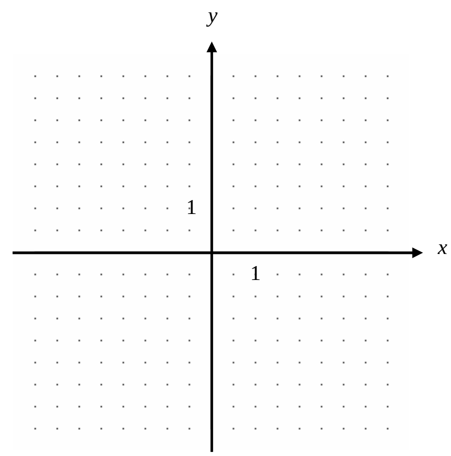
a) $\frac{dy}{dx} = x$

b) $\frac{dy}{dx} = y$

c) $\frac{dy}{dx} = -\frac{x}{y}$







Example: Suppose that you know that the point given below is on a particular solution of the differential equation given in the last example. By following slopes, draw on the diagrams above what you think the particular solution look like. (♬: The graph should follow the pattern of the slope field, but may go between the points rather than through them.)

a) $(-2, -1)$

b) $(0, 0.5)$

c) $(2, 2)$

Example: Solve the differential equations from the previous example by first **separating the variables**. Find the particular solution that contains the point given in the last example. Does your solution make sense when compared to the graph of the slope field?

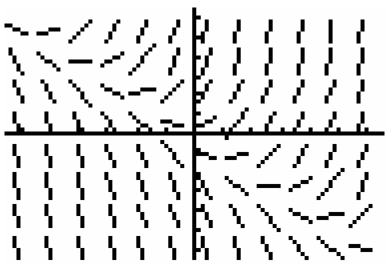
a) $\frac{dy}{dx} = x$

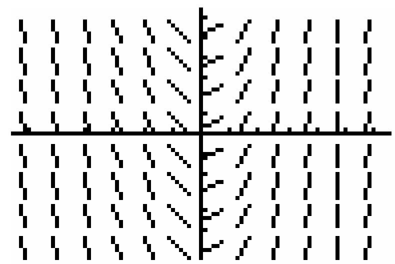
b) $\frac{dy}{dx} = y$

c) $\frac{dy}{dx} = -\frac{x}{y}$

Example: Solve the initial value problem $\frac{d^2y}{dx^2} = 2 - 6x$ given that $y(0) = 1$ and $y'(0) = 4$.

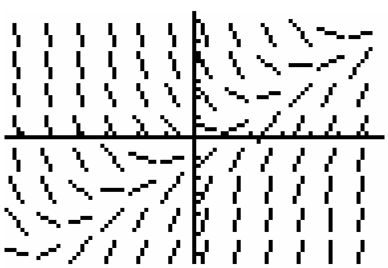
Example: Match the six slope fields shown below to their differential equations. Explain each choice.

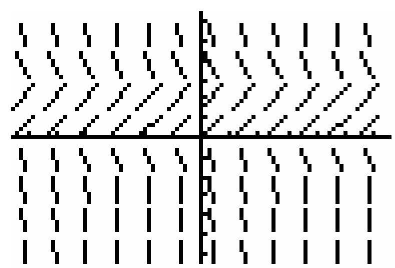




$$\frac{dy}{dx} = x - y$$

$$\frac{dy}{dx} = 2x$$

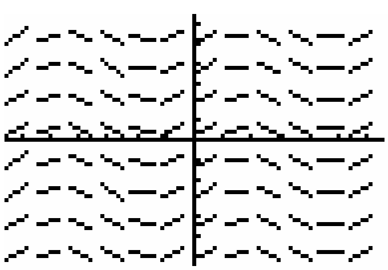


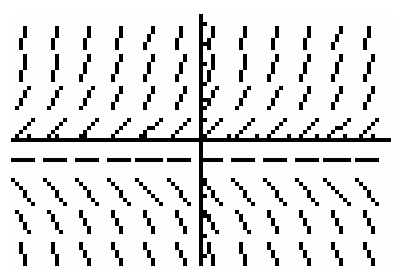


$$\frac{dy}{dx} = 1 + y$$

$$\frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} = x + y$$





$$\frac{dy}{dx} = y(3 - y)$$