1.1 LINES

Much of Calculus focuses on the concept of "local linearity", meaning that even if a function curves, if you were to pick a point and stay very close (local) to that point, the function behaves very much like that of a line.

Example: Graph the functions $y = \sin x$ and $y = x$ on your calculator. Obviously these are not the same function. However, if you were to stay close to the point (0, 0), these two functions are very close. To see this, use the ZOOM feature of your calculator, and zoom in on (0, 0). Try zooming in more than once.

We can say that as long as we stay "close" to (0, 0), the functions $y = \sin x$ and $y = x$ are almost the same thing. Now, the concept of "close" is more complicated than it might sound, but more on that in chapter 2. For now, we focus on lines.

As stated in the syllabus, calculus has to do with change. For notational purposes, we use the capital Greek letter delta, Δ.

Slope

The slope of a non-vertical line is given by

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

A vertical line has ________________________, and a horizontal line has ________________________ .

Parallel Lines have slopes that are __________________ .

Perpendicular Lines have slopes that are ______________________________________.

IMPORTANT ♬: You will be best served in calculus if you think of slope as a _________________________________.

Equations of a Line

The first equation of a line you used in algebra was probably the *slope – intercept form*: ___________________________

The slope is ______, and the *y*-intercept is ________.

In calculus, it is actually easier to write the equation of a line in *point – slope form*: _______________________________

The point is ________, and the slope is __________.

♬: To write an equation of a line, all you need is a ___________________ and the _________________.

Another format used to write the equation of a line is called *standard (general) form*: ____________________________

Example: Which of the equations above has "*y* written as a function of *x*" ?

Example: The point-slope form is written as _____________________________ if you want "*y* written as a function of *x*"

Example: Find the equations of the lines passing through (–2, 4) and having the following characteristics:

a) Slope of $\frac{7}{16}$

b) Parallel to the line $5x - 3y = 3$

c) Passing through the origin

d) Parallel to the y – axis.

Example: Find the equations of the lines passing through (1, 3) and having the following characteristics:

a) Slope of $-\frac{2}{3}$

b) Perpendicular to the line $x + y = 0$

c) Passing through the point (2, 4)

d) Parallel to the x – axis.

Example: [Page 11, #57] Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4).

Example: The relationship between Fahrenheit and Celsius temperatures in linear.

a) Use the facts that water freezes at 0° C or 32° F, and water boils at 100° C or 212° F (not your recollection of temperature formulas) to find an equation that relates Celsius and Fahrenheit.

b) Using your equation, find the Celsius equivalent of 80° F and the Fahrenheit equivalent of –10° C.

c) [Page 11, #54] Is there a temperature at which a Fahrenheit thermometer and a Celsius thermometer give the same reading? If so, what is it?

[see part *b* in the textbook for a hint]

Regression Analysis is a process of finding a curve to fit a set of data. The basic process involves plotting the points and finding a function that “best fits” those points. The curve you find is called the regression curve. For the purposes of this section, our “curve” is linear, but it could be a parabola or other power function, a logarithmic function, a trigonometric function, or an exponential function.

Example: [Page 11, #53] The median price of existing single-family homes has increased consistently during the past few years. However, the data in the table below show that there have been differences in various parts of the country.

Year	South (dollars)	West (dollars)
1999	145,900	173,700
2000	148,000	196,400
2001	155,400	213,600
2002	163,400	238,500
2003	168,100	260,900

a) Find the linear regression equation for home cost in the South.

b) What does the slope of the regression line represent?

c) Find the linear regression equation for home cost in the West.

d) Where is the median price increasing more rapidly, in the South or the West?

Notecards from Section 1.1: Rules for Rounding

1.2 FUNCTIONS AND GRAPHS

Functions

In the last section we discussed lines and when we needed to write "y as a function of x". But what is a function?
In Algebra 1, we defined a function as a rule that assigned one and only one (a unique) output for every input. We called the input the *domain* and the output the *range*.

Definition: Function

A **function** from a set D to a set R is a rule that assigns a unique element in R to each element in D.

Intervals

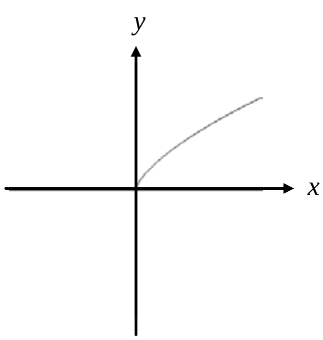
In this course we would like not only to know what the domain and range are, but how to describe them with the correct notation. The domain and range of a function could be all real numbers, or we may need to limit the domain and/or the range using *intervals* that are either *open* or *closed*.

Open and closed intervals have endpoints. If the **endpoint is included**, then we say the interval is **closed** at that point, and if the **endpoint is not included,** then we say the interval is **open** at that point. We use a parenthesis to indicate open and a bracket to indicate closed.

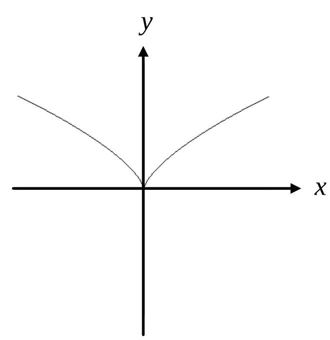
Example: Use interval notation AND inequality notation to describe each interval on the x – axes below.

Example: What are some things to look for in a function that might restrict the domain.

Example: Using the computer program Derive5, I graphed the equations below. What do you notice?



$y = x^{2/3}$



$y = \left(x^2\right)^{1/3}$

The purpose of the last example is for you to understand that while the calculator is a wonderful tool, it is not always perfect. For the purpose of this last example, your TI-83+ calculator will graph the function correctly either way. I personally find this interesting, since both the TI-83+ and the program Derive5 are produced by Texas Instruments.

**You should ALWAYS be aware of the domain of a function … especially when dealing with applications.

♬: While it is many times possible to just look for the restrictions to the domain, the range of a function is easier to determine if you have a graph or you know what the graph looks like. You should know what the following basic functions look like without having to use your calculator. Can you draw a quick sketch?

$y = x$	$y = x^2$	$y = x^3$	$y = \sqrt{x}$
$y = \dfrac{1}{x}$		$y = \dfrac{1}{x^2}$	
$y = \lvert x \rvert$	$y = \lfloor x \rfloor$	$y = ab^x$, $0 < b < 1$	$y = ab^x$, $1 < b$
$y = \log x$	$y = \sin x$	$y = \cos x$	$y = \tan x$

Even and Odd Functions

Recognizing the behavior of functions is not limited to their domain and range. Many functions have the symmetric property of being odd or even. You need to be able to recognize the graph of a function as odd or even, AND you need to understand how to show/verify/prove that a function is even or odd algebraically.

Graphical Recognition of Even and Odd Functions

An **EVEN** function is symmetrical about the *y* – axis. *Example*: *y* = cos *x*

An **ODD** function is symmetrical about the origin. *Example*: *y* = sin *x*

Algebraic Properties of Even and Odd Functions

An **EVEN** function has the property that $f(-x) = f(x)$.
That is, if you plug in "–*x*" into the function and simplify, you will obtain the original function.

An **ODD** function has the property that $f(-x) = -f(x)$.
That is, if you plug in "–*x*" into the function and simplify, you will obtain the opposite of the original function.

Example: Prove whether the following functions are even, odd, or neither.

a) $g(x) = x^3 - x$

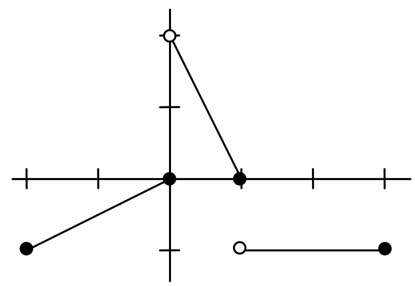
b) $h(x) = 1 + \cos x$

Piecewise Functions

Some functions are broken into pieces and behave differently depending on the restricted domain of each piece. Such functions are called piecewise functions. An example of a function that can be written as a piecewise function is the absolute value function $f(x)=|x|$.

Example: Sketch a picture of $f(x)=|x|$ and write an equation for the two "pieces" using a domain appropriate to each piece.

Example: Write a piecewise function for the graph at the right.



Composite Functions

When the range of one function is used as the domain of a second function we call the entire function a composite function. We use the notation $(f \circ g)(x)= f(g(x))$ to describe composite functions.
This is read as "*f* composed with *g*" or "*f* of *g* of *x*".

Example: If $f(x)=1-x^2$ and $g(x)=\sqrt{x}$, find $g(f(x))$. What is the domain and range of $g(f(x))$?

Example: If $f(x)=\dfrac{2x-1}{x+3}$, and $g(x)=\dfrac{3x+1}{2-x}$, find $f(g(x))$. Based on your answer, how might f and g be related?

Notecards from Section 1.2: Odd/Even Functions

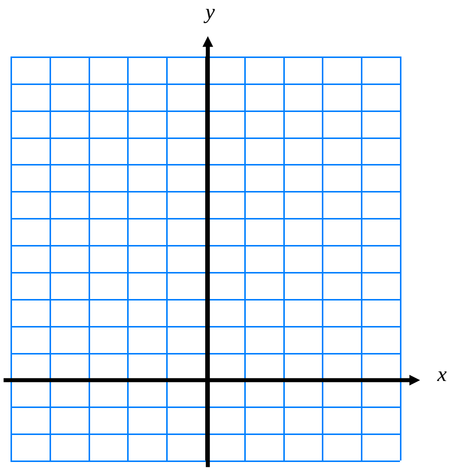
1.3 EXPONENTIAL FUNCTIONS

So far we've dealt with linear functions, piecewise functions, and composite functions. Next up, exponential functions.

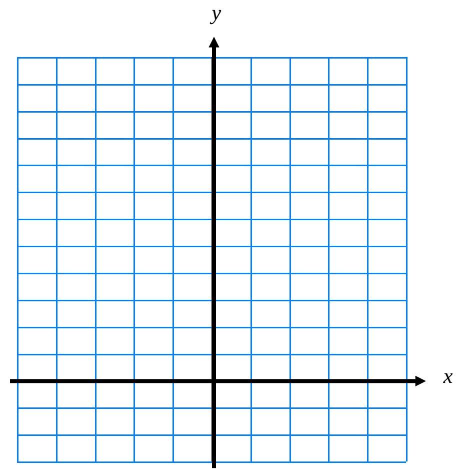
> *Definition*: *Exponential Function*
>
> If $b > 0$ and $b \neq 1$, then $f(x) = b^x$ is an exponential function with base *b*.

Example: Sketch the following graphs as accurately as possible on the graphs below:

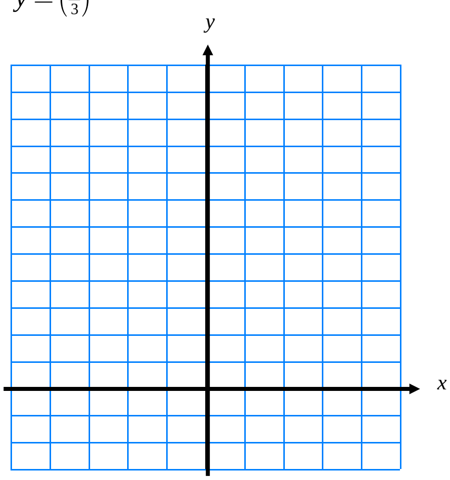
a) $y = 3^x$



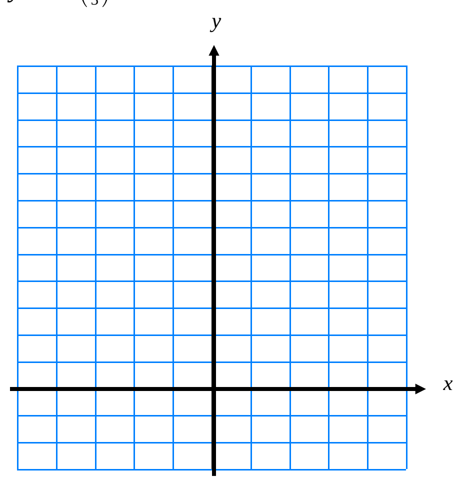
b) $y = 2 \cdot 3^x$



c) $y = \left(\frac{1}{3}\right)^x$



d) $y = 2 \cdot \left(\frac{1}{3}\right)^x$



Example: Which of these graphs show growth? decay?

Example: How does the 2 affect the graph?

Example: What is the domain and range of all 4 graphs?

Exponential Growth/Decay Model

In the exponential model $y = a \cdot b^x$, *b* is the rate of growth if *b* > 1, and *b* is the rate of decay if 0 < *b* < 1.
In either case, the initial value is *a*.

Example: Using your graphing calculator, let $Y_1 = x^2$ and $Y_2 = 2^x$. Graph both equations in the same window.

a) Solve the equation $x^2 = 2^x$ using your graphing calculator. Where are the solutions to this equation and how many are there?

b) Clear the two graphs from the screen and use the equation $x^2 - 2^x = 0$. Solve for *x* be graphing the left side of this equation. Where are the solutions to this equation and how many are there?

c) What did you learn from the last two questions?

The Number e

Many exponential functions in the real world are modeled using the base of *e*. Just like $\pi \approx 3.14$, we say $e \approx 2.718$. We can also define *e* using the function $\left(1+\frac{1}{x}\right)^x$ as follows:

$$\text{As } x \to \infty, \ \left(1+\tfrac{1}{x}\right)^x \to e$$

Example: Suppose you invest $12,000 in an account that earns you 5% interest compounded monthly for 10 years.

a) What is the initial amount?

b) What is the growth rate?

c) How many times does your money grow in 10 years? (How many times is interest added to your account?)

d) How much money will you have in 10 years?

e) Suppose the interest was compounded continuously. How much would you have in 10 years?

Example: You buy a brand new car for $35,000 and find out it depreciates at 12.5% per year. Write an exponential equation modeling this situation. How much will your car be worth in 5 years?

Example: The half – life of Ra – 226 is 1,620 years. If there are 10g initially, how much Ra – 226 is left after 1,000 years?

Example: The number of United States citizens y (in millions) who traveled to foreign countries in the years 1988 through 1996 are shown in the table below., where $t = 8$ represents the year 1988.

t	8	9	10	11	12	13	14	15	16
y	40.7	41.1	44.6	41.6	43.9	44.4	46.5	50.8	52.3

a) Use the regression capabilities of your graphing calculator to find an exponential model that fits the data.

b) According to the model, is the number of travelers increasing or decreasing? At what rate?

c) Using your model, how many travelers were there in 1980? 1974? 2006?

d) Why is it important to let $t = 8$ represent the year 1988? [Try answering question *c* using the actual year.]

1.4 PARAMETRIC EQUATIONS

Up to this point all the functions we have been looking at have used a single equation with two variables, *x* and *y*. In this section we use a third variable to represent the curve. This third variable is called a parameter.

Example: Using your graphing calculator, set your window to $X_{\min} = -5, \quad X_{\max} = 80, \quad X_{\text{scl}} = 5,$
$Y_{\min} = -5, \quad Y_{\max} = 20, \quad Y_{\text{scl}} = 5.$

Graph $y = -\dfrac{x^2}{72} + x$, which models the path of an object thrown into the air at a 45° angle at an initial velocity of 48 feet per second. (Just take my word for it ☺)

Use [TRACE] to locate a few points on the graph. What do you learn about the object from this information?

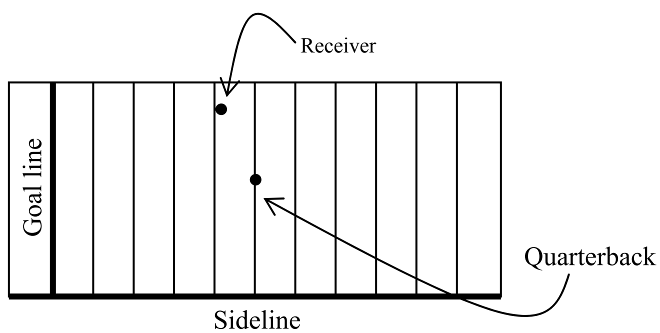
Example: Now change your calculator to *parametric* mode and plot $x_1 = 24\sqrt{2}t$.
$y_1 = -16t^2 + 24\sqrt{2}t$

Notice it is the same graph … more on that later.

Use [TRACE] to locate a few points on the graph. What do you learn about the object from this information that you did not know before?

Example: During a football game, the quarterback held the ball on the 50 yard line while the receiver ran toward the goal line. After waiting 3 seconds, the quarterback threw the ball to the receiver.

Using the intersection of the goal line and the sideline as the origin, let *x* = the number of yards from the goal line, *y* = the number of yards from the sideline, and *t* = the number of seconds the receiver has been running.



The two equations describing the receiver's path are given by $x_1 = 42 - 6t \qquad 0 \le t \le 7.5$
$y_1 = 45 - t$

The two equations describing the path of the ball are given by $x_2 = 50 - 22(t-3) \qquad 3 \le t \le 7.5$
$y_2 = 27 + 6(t-3)$

**These equations give the path of the ball viewed from above. They ignore the height of the ball.

a) Choose an appropriate window and graph the receiver's path. Explain your choice for the window.

b) Graph the path of the ball.

♪: To make sure that the graph doesn't start for 3 seconds, enter $x_2 = 50 - 22(t-3)(t \ge 3)$
$y_2 = 27 + 6(t-3)(t \ge 3)$

c) Assuming the height of the ball is not an issue, does the receiver catch the ball? Explain your reasoning. If not, change the equations above so that the receiver does catch the ball.

Definition: *Parametric Curve*

If x and y are given as functions

$$x = f(t), \quad y = g(t)$$

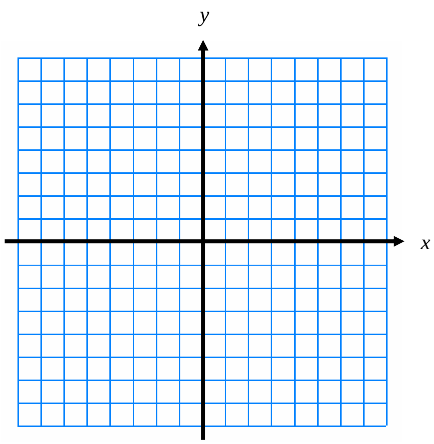
over an interval of t values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve.

Graphing Parametric Curves Without a Calculator

Just like when you learned to graph for the first time back in Algebra 1, we are going to make a table of values. The difference is that we now have three variables instead of two.

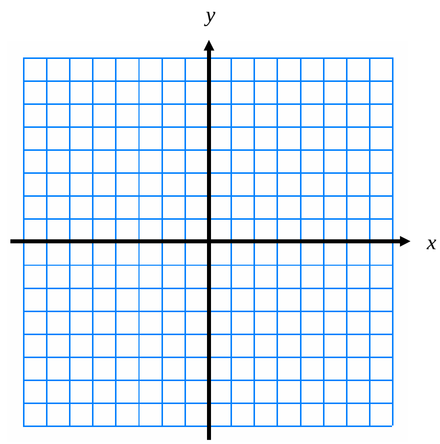
Example: Graph the parametric curve

$$x = t^2 - 4 \qquad -2 \le t \le 3$$
$$y = \frac{t}{2}$$



Example: Graph the parametric curve

$$x = 4t^2 - 4 \qquad -1 \le t \le \tfrac{3}{2}$$
$$y = t$$



Example: Compare and contrast the two graphs above.

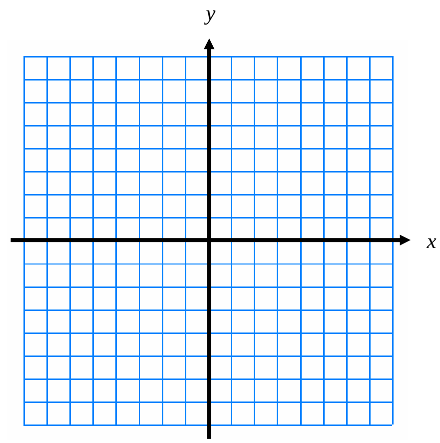
Changing from Parametric to Rectangular (Cartesian)

To change a parametric equation back into a more familiar rectangular (Cartesian) equation you must eliminate the parameter. The typical approach to doing this is to solve for the parameter in one of the equations and then simply substitute that solution into the other equation.

Example: Change the parametric equation defined by $x = \sqrt{t}$, $y = 2 - t$ into a Cartesian equation.

The next two examples illustrate another way to eliminate the parameter by using trigonometric identities.

Example: Graph $\begin{aligned} x &= 3\cos\theta + 2 \\ y &= 5\sin\theta - 1 \end{aligned}$ for $0 \le \theta \le 2\pi$, then change the parametric equation into a Cartesian equation.



Example: Change the parametric equation defined by $\begin{aligned} x &= 4\sec\theta \\ y &= 3\tan\theta \end{aligned}$ into a Cartesian equation. Can you identify the graph of this equation without graphing it?

Writing a Parametric Equation

Changing from Rectangular to Parametric means you get to create a parameter. As the graphs on the previous page indicated, your choice of parameter should not change the shape of the graph, only the "speed" in which the graph is drawn.

Example: Find a parametrization for the left half of the parabola $y = x^2 + 2x$.

Example: Find a parametrization for the line segment with endpoints (–1, 3) and (3, –2).

PARENT FUNCTIONS AND TRANSFORMATIONS

Parent Functions

One of the things we do in calculus is study the behavior of functions. Some of the most basic functions you should be able to recognize and graph without the use of a calculator. You should be able to sketch a fairly accurate graph of the following parent functions:

$y = x$	$y = x^2$	$y = x^3$	$y = \sqrt{x}$
$y = \frac{1}{x}$		$y = \frac{1}{x^2}$	
$y = \lvert x \rvert$	$y = \lfloor x \rfloor$	$y = ab^x$, $0 < b < 1$	$y = ab^x$, $1 < b$
$y = \log x$	$y = \sin x$	$y = \cos x$	$y = \tan x$

Transformations

Not only should you be able to graph the parent functions above, but you should be able to graph the transformations of these graphs.

Example: Suppose you are given the function $f(x)$. What effect do *a*, *b*, *c*, and *d* have on original function if your new function is $a \cdot f(b(x+c)) + d$

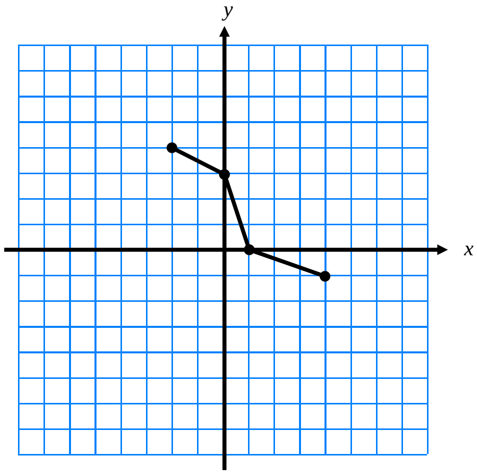
Another way to look at this is with the following chart:

	Inside		Outside	
$+/-$	$f(x+c)$	$f(x-c)$	$f(x)+d$	$f(x)-d$
$\times/\div$	$f(bx)$	$f(-x)$	$a \cdot f(x)$	$-f(x)$

Example: Let f be the graph given in the picture below. Graph the following transformations of f.

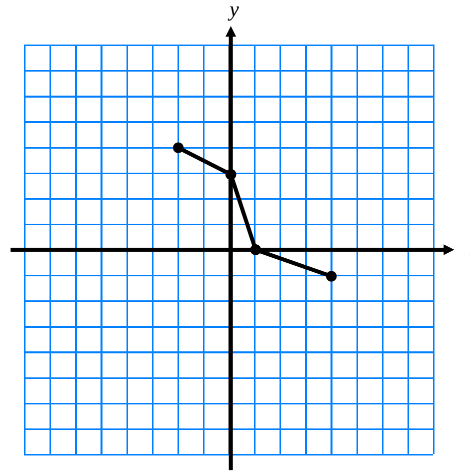
a) $f(x)+2$

b) $-f(x)$



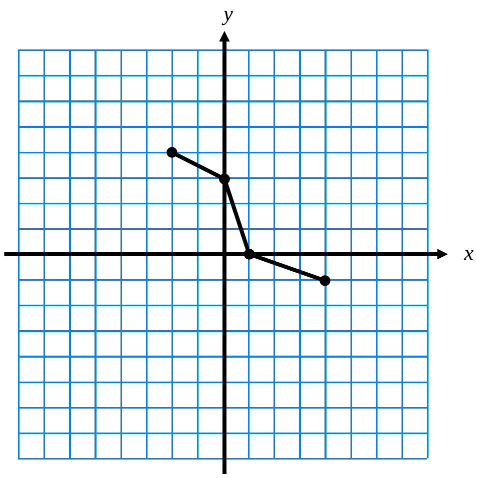
c) $f(x-2)$

d) $f(x+3)$



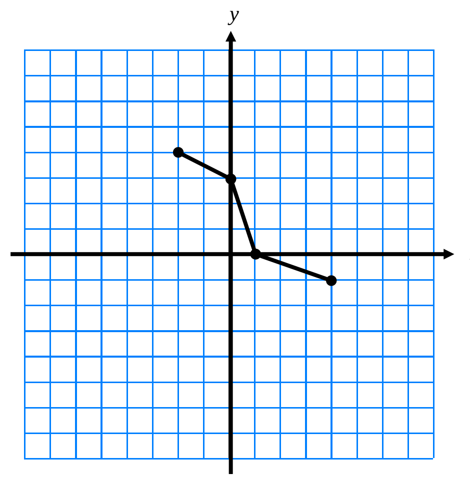
e) $2f(x)$

f) $f(-x)$



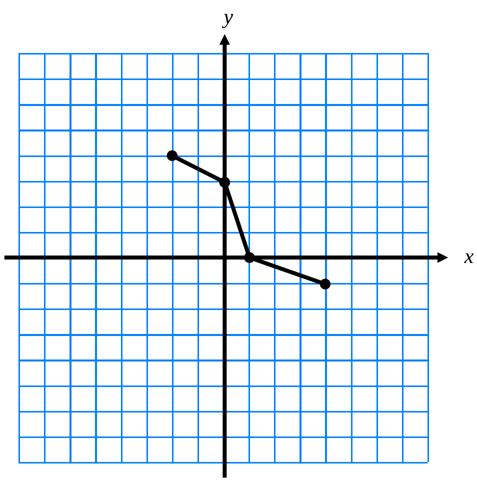
g) $f\left(\frac{1}{2}x\right)$

h) $f(2x-6)$

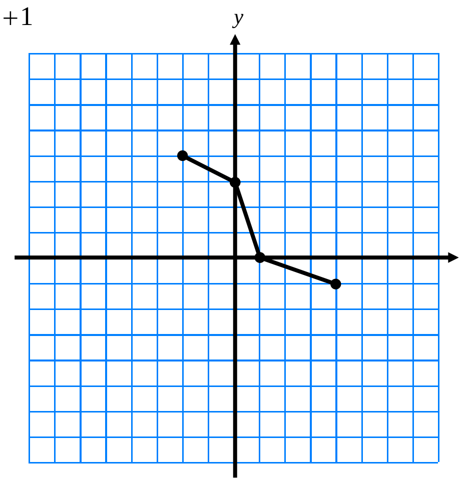


i) $f(|x|)$

j) $|f(x)|$



k) $\frac{1}{2}f(3x-9)+1$



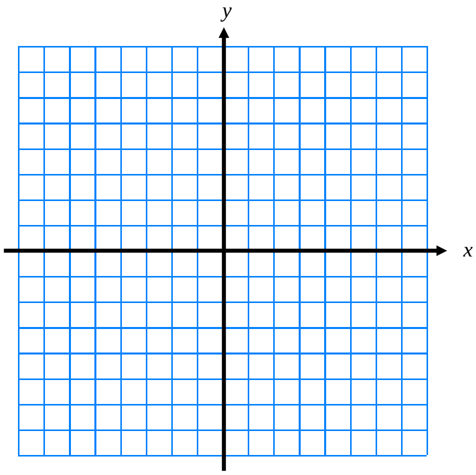
Example: First graph $f(x)=\log_2 x$. Then graph the following transformations:

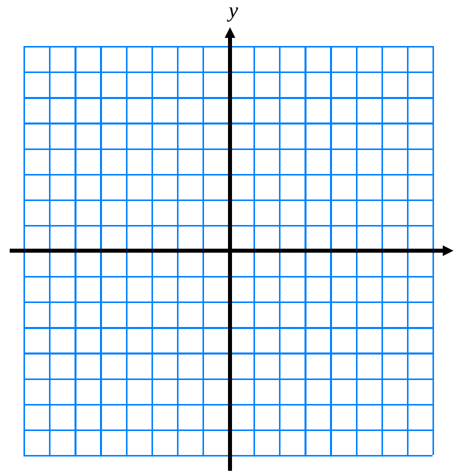
a) $y=4-f(x)$

b) $y=-f(x-1)$

c) $y=\frac{1}{4}f(x+2)$

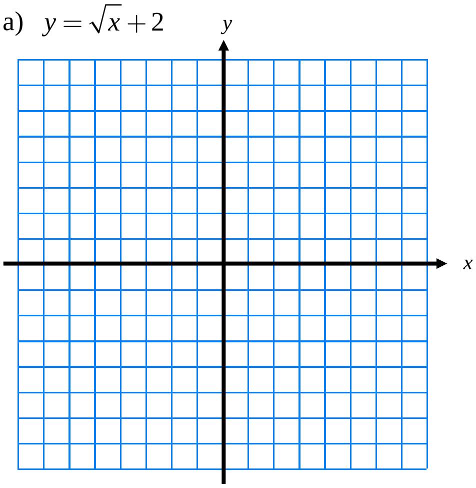
d) $y=-2f(x-1)+3$



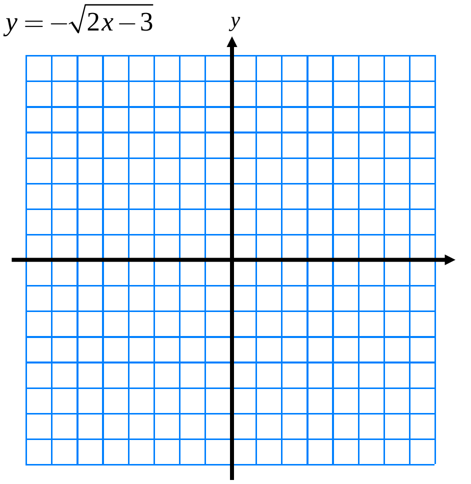


Example: For *a* – *c*, graph $f(x)=\sqrt{x}$ first, and then graph each transformation.

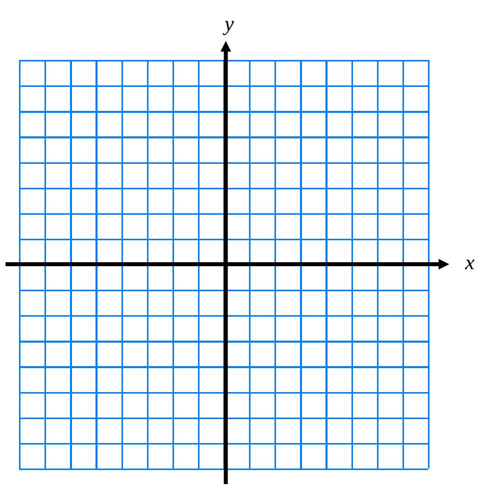
a) $y=\sqrt{x}+2$



b) $y=-\sqrt{2x-3}$



c) $y=2\sqrt{3x+2}-1$



Notecards from Parent Functions and Transformations: Transformations (Parent Functions)

1.5 FUNCTIONS AND LOGARITHMS

Inverse Functions

In technical jargon, an inverse of a function maps the elements of the range to the elements of the domain. In English, this means that the inverse of a function reverses the domain and range. Not all graphs were defined as functions, and we had the *vertical line test* to determine whether a graph was or was not a function. Similarly, not all functions have an inverse, and we have the *horizontal line test* to determine whether or not a function has an inverse.

Definition: *One – to – One Function*

A function $f(x)$ is **one – to – one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.
A function that is one – to – one has an inverse.

The definition above can be seen graphically with the use of a horizontal line test. If there are two x – values for any given y – value of function, then the function does NOT have an inverse.

Example: Does $y = x^2 + 5x$ have an inverse? Why or why not?

Example: Does $y = x^3 + x$ have an inverse? Why or why not?

Once we know whether a function has an inverse, our next task is to find an equation and/or a graph for the inverse.

Finding the Inverse Graphically (two ways)

1. Reflect the graph of the original function over the line $y = x$.
2. Plot the reverse of the coordinates.

Finding the Inverse Algebraically

Switch the x and y in the original equation, then solve the new equation for y in order to write y as a function of x.

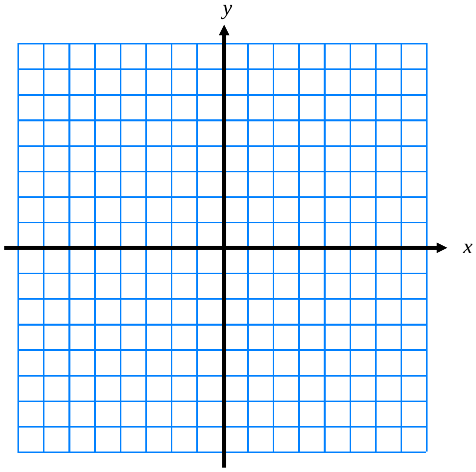
Example: Let $f(x) = x^3 - 1$.

a) Graph the function on the grid to the right.

b) Draw the line $y = x$

c) Reflect the graph of $f(x)$ over the line $y = x$.

d) Find the inverse of the function algebraically.



e) Use your graphing calculator to verify your answer to part *d*.

Verifying Inverses

It is one thing to find the inverse function (either graphically or algebraically), but it is another to verify that two functions are actually inverses. Whenever you are verifying anything in mathematics, you must go back and use the definition.

Definition: Inverse Function

A function $f(x)$ has an inverse $f^{-1}(x)$ if and only if $f\left(f^{-1}(x)\right) = x = f^{-1}\left(f(x)\right)$

Example: According to this definition, how many composite functions must you use to check whether or not two functions are inverses of each other?

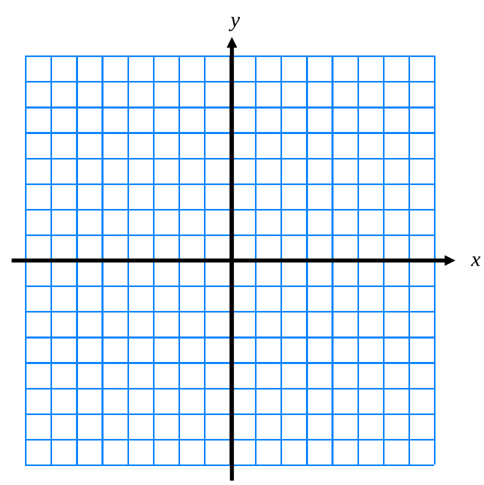
Example: Find $f^{-1}(x)$ and verify if $f(x) = \dfrac{x+3}{x-2}$.

Logarithmic Functions

How do Logarithms fit into this discussion? A logarithmic function is just the inverse of an exponential function.

Example: Graph $y = 2^x$ and find the inverse of the function graphically.

The equation of the inverse function is ________________.



Properties of Logarithms

Definition of a logarithm:

$$\log_a x = y \iff a^y = x$$

Inverse Properties of a logarithm:

$$a^{\log_a x} = x \qquad \log_a a^x = x$$

Logarithm of a product:

$$\log_a (xy) = \log_a x + \log_a y$$

Logarithm of a power:

$$\log_a \left(x^y\right) = y \cdot \log_a x$$

Logarithm of a quotient:

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Change of Base Formula:

$$\log_a x = \frac{\ln x}{\ln a}$$

Other properties:

$$\log_a a = 1 \qquad \log_a 1 = 0$$

Example: Evaluate the following without using your calculator.

a) $\log_2 \frac{1}{8} =$

b) $\log_{27} 9 =$

Example: Solve for x in the following equations.

a) $3^{2x} = 75$

b) $3\left(5^{x-1}\right) = 86$

c) $e^{4x} = 15$

d) $\log_2 (x-1) = 5$

e) $\ln \sqrt{x+2} = 1$

f) $\log(8x) - \log\left(1+\sqrt{x}\right) = 2$

Example: The population of Glenbrook is 375,000 and is increasing at the rate of 2.25% per year. Predict when the population will be 1 million.

Example: Using the properties of logarithms, write *y* as a function of *x*.

$$\ln(y-1)-\ln 2 = x+\ln x$$

Example: Solve for *x*: $2^x + 2^{-x} = 5$

Example: The table below shows the amount of Canadian Oil Production since 1960.

Year	**Metric Tons (millions)**
1960	27.48
1970	69.95
1990	92.24

a) Find a natural logarithm regression equation for the data in the table.

b) Estimate the number of metric tons of oil produced by Canada in 1985.

c) Predict when Canadian oil production will reach 120 metric tons.

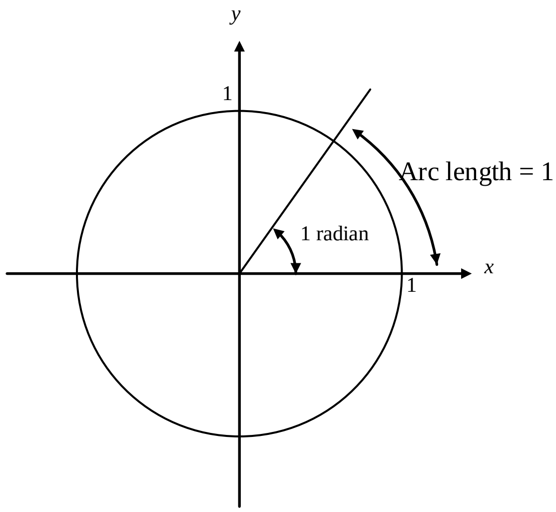
Notecards from Section 1.5: Inverse Functions

1.6 TRIGONOMETRIC FUNCTIONS

There are two common measures of angles: degrees and radians. As you'll see, almost all of calculus uses radians. Before beginning any exercise with trigonometric functions, make sure your calculator is set in *radian* mode. (ESPECIALLY THOSE YOU WHO HAD PHYSICS YESTERDAY!) Unless otherwise stated, the angles in the text are measured in radians. For example, sin 3 means the sine of 3 radians, but sin 3° means the sine of 3 degrees. Just for fun, you should understand exactly what a radian is.

Definition: Radian

An angle of **1 radian** is defined to be the angle at the center of a unit circle which spans an arc of length 1, measured counterclockwise.

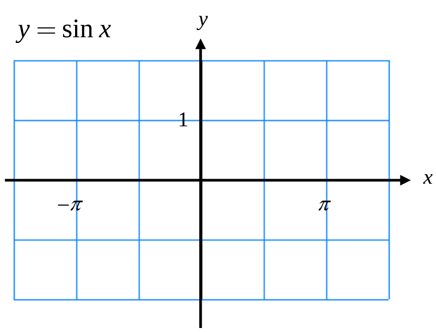


♬: You should also be VERY familiar with the 6 trigonometric values of the key points on the unit circle $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$

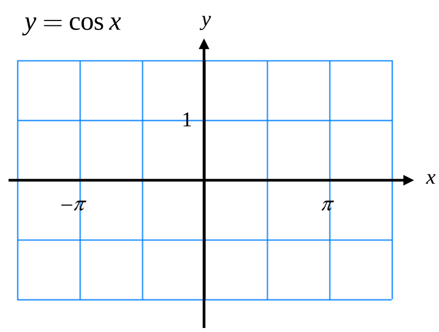
Graphs of Trigonometric Functions

Graph all 6 trigonometric functions below. Include both positive and negative values of *x*.

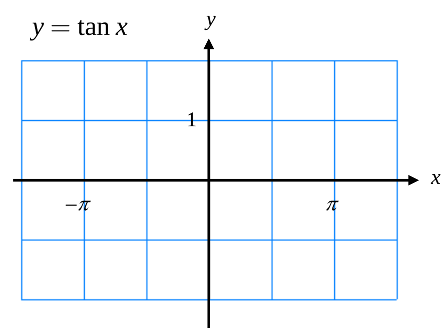
$y = \sin x$



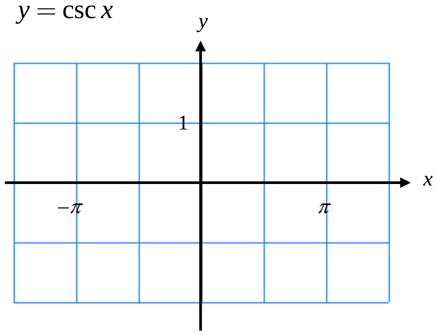
$y = \cos x$



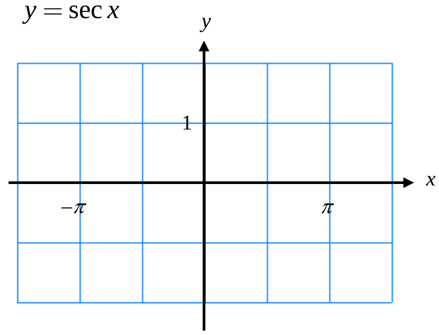
$y = \tan x$



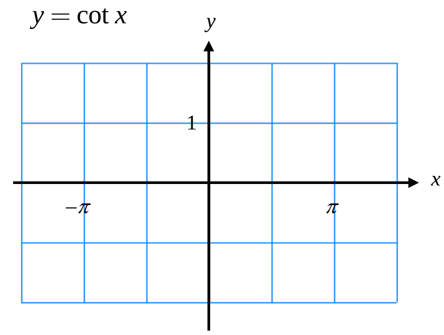
$y = \csc x$



$y = \sec x$



$y = \cot x$

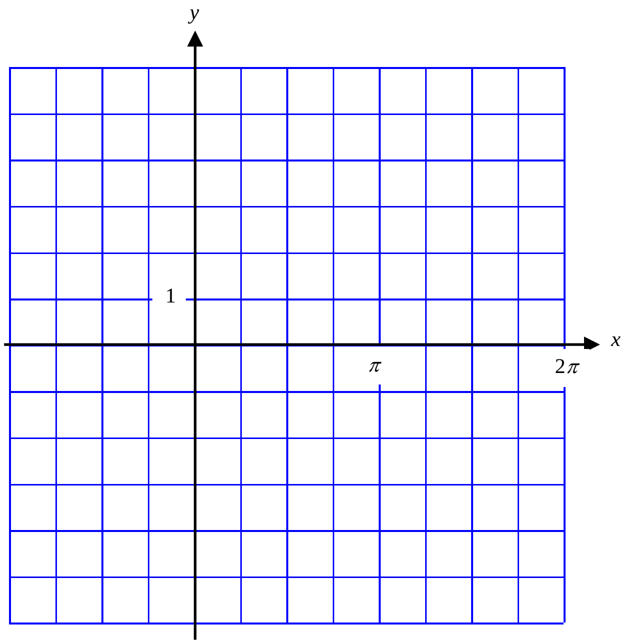


Transformations of Trigonometric Functions

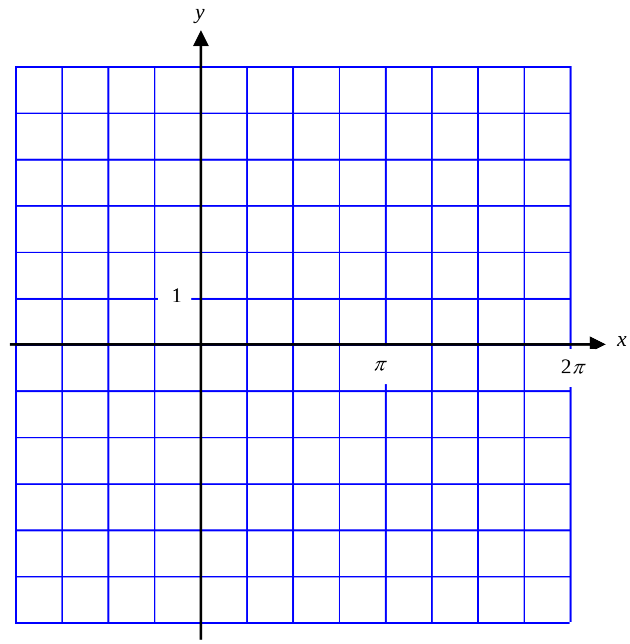
Example: For each of the following examples, do three things.

1. Describe the transformation.
2. Graph the function.
3. Where appropriate, give the amplitude and the period.

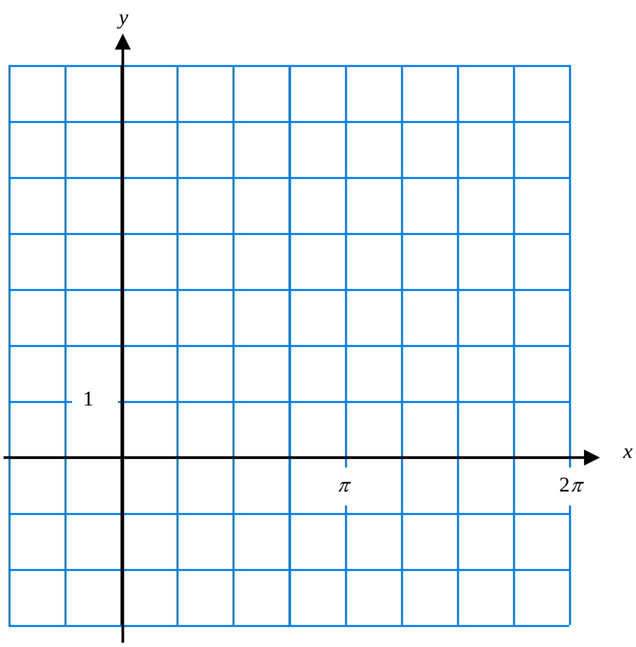
a) $y = 2\cos x - 3$



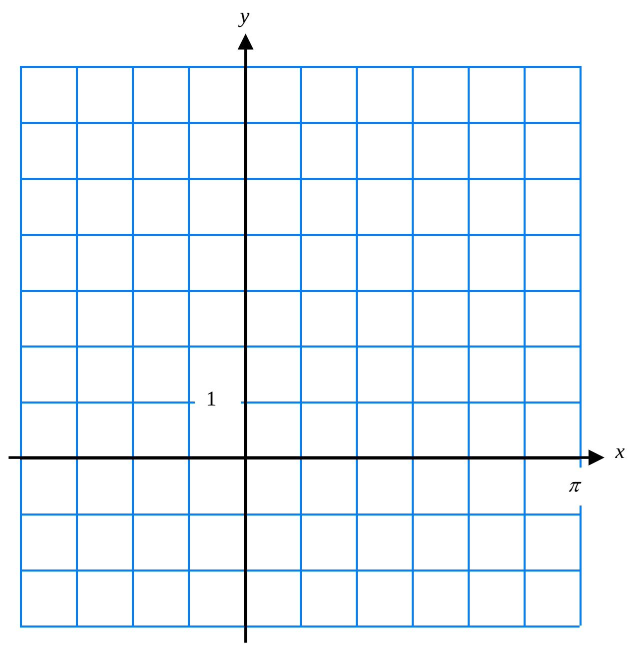
b) $y = \sin(-2x + \pi)$



c) $y = 2\sin(4x + \pi) + 3$



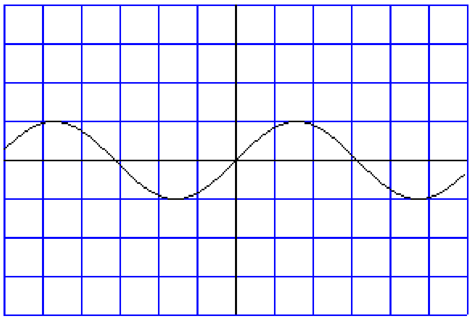
d) $y = -2\tan(3x - \pi) + 1$



Inverse Trigonometric Functions

Due to the periodic property of trigonometric functions, all 6 of the trig functions fail the horizontal line test (test for whether or not an inverse exists). In the graphs below, color (or highlight) the appropriate part of the trig function to use for its inverse, then graph it's inverse function.
Each box on each grid represents 1. You may wish to put appropriate values of π on the grid.

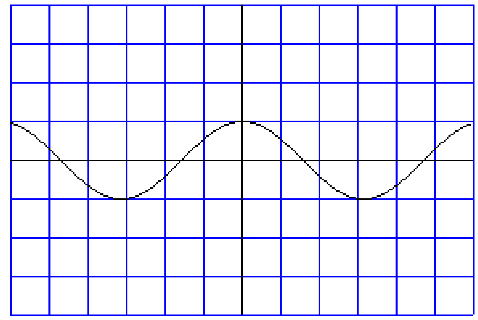
$y = \sin x$



Domain of Inverse:

Range of Inverse:

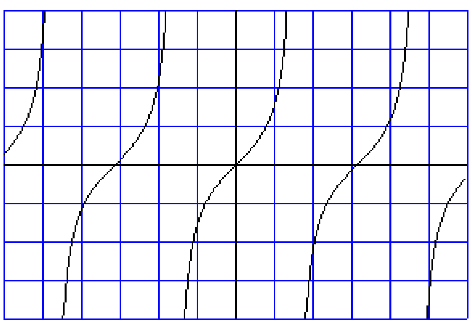
$y = \cos x$



Domain of Inverse:

Range of Inverse:

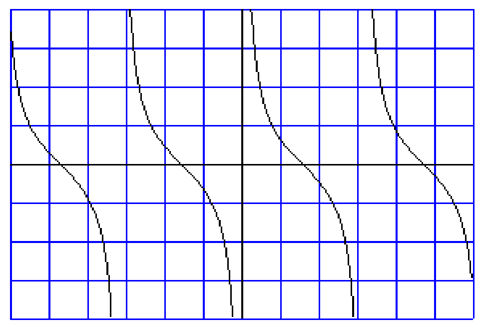
$y = \tan x$



Domain of Inverse:

Range of Inverse:

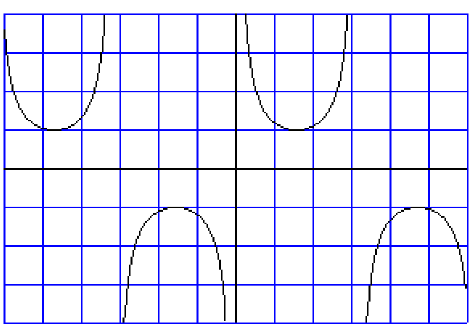
$y = \cot x$



Domain of Inverse:

Range of Inverse:

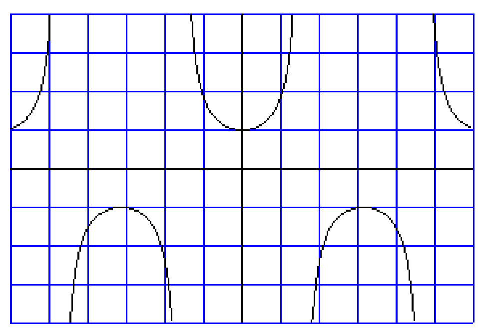
$y = \csc x$



Domain of Inverse:

Range of Inverse:

$y = \sec x$



Domain of Inverse:

Range of Inverse:

The restricted domain of the inverse trig functions means you must pay close attention to your solutions. Your calculator only gives you the solution for which the domain of the inverse function is defined. Your calculator also only has 3 of the six inverse functions.

Example: Find the domain and range of the following functions:

a) $\sin\left(\cos^{-1}(x)\right)$

b) $\sec\left(\tan^{-1}(x)\right)$

Example: Evaluate the expression WITHOUT a calculator. $\tan\left(\sin^{-1}\left(\frac{12}{13}\right)\right)$

Example: Solve for *x*: $\sec x = 3$ where $0 \le x \le 2\pi$.

The next two examples came from the 2007 AP Exam Free Response #4 (without a calculator). While the question itself focused on topics we will not cover until later in the year, the problems students had in answering the question stemmed from solving the following equations.

Example: Solve for t if $0 \le t \le 2\pi$. $e^{-t}\cos t + \sin t\left(-e^{-t}\right) = 0$

Example: Solve for A: $A\left(-2e^{-t}\cos t\right) + e^{-t}\left(\cos t - \sin t\right) + e^{-t}\sin t = 0$

Notecards from Section 1.6: Trigonometric Identities